1 When plugging in works.

Consider the function $f(x) = x^2$, and suppose we wish to find the following limits:

(a) $\lim\limits_{x \to 2^+} f(x)$

(b) $\lim\limits_{x \to 2^-} f(x)$

(c) $\lim\limits_{x \to 2} f(x)$.

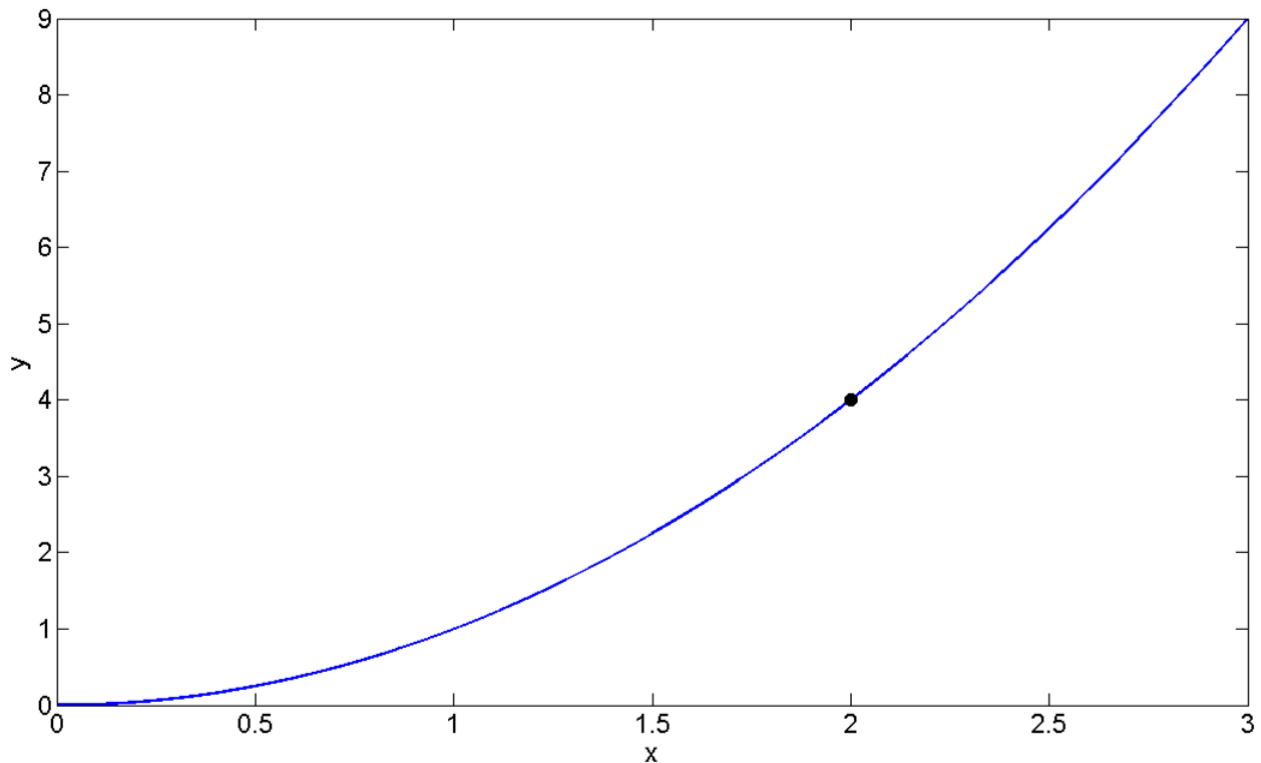Here is a plot of the function $f(x) = x^2$ on the interval $[0, 3]$.

Figure 1: *A plot of the function $f(x) = x^2$. The closed circle is at the point $(2, 4)$.*

Now, we can determine the limits by "just plugging in" the value 2 into the function $f(x) = x^2$ and we obtain:

1

(a) $\displaystyle\lim_{x \to 2+} x^2 = 2^2 = 4$

(b) $\displaystyle\lim_{x \to 2^-} x^2 = 2^2 = 4$

(c) $\displaystyle\lim_{x \to 2} x^2 = 2^2 = 4$

We can also see all of these by looking at the graph.

2   When plugging in does not work.

Consider the floor function we studied in class today, $g(x) = \lfloor x \rfloor$. Here is a graph of the floor function on the interval $[0, 3]$.
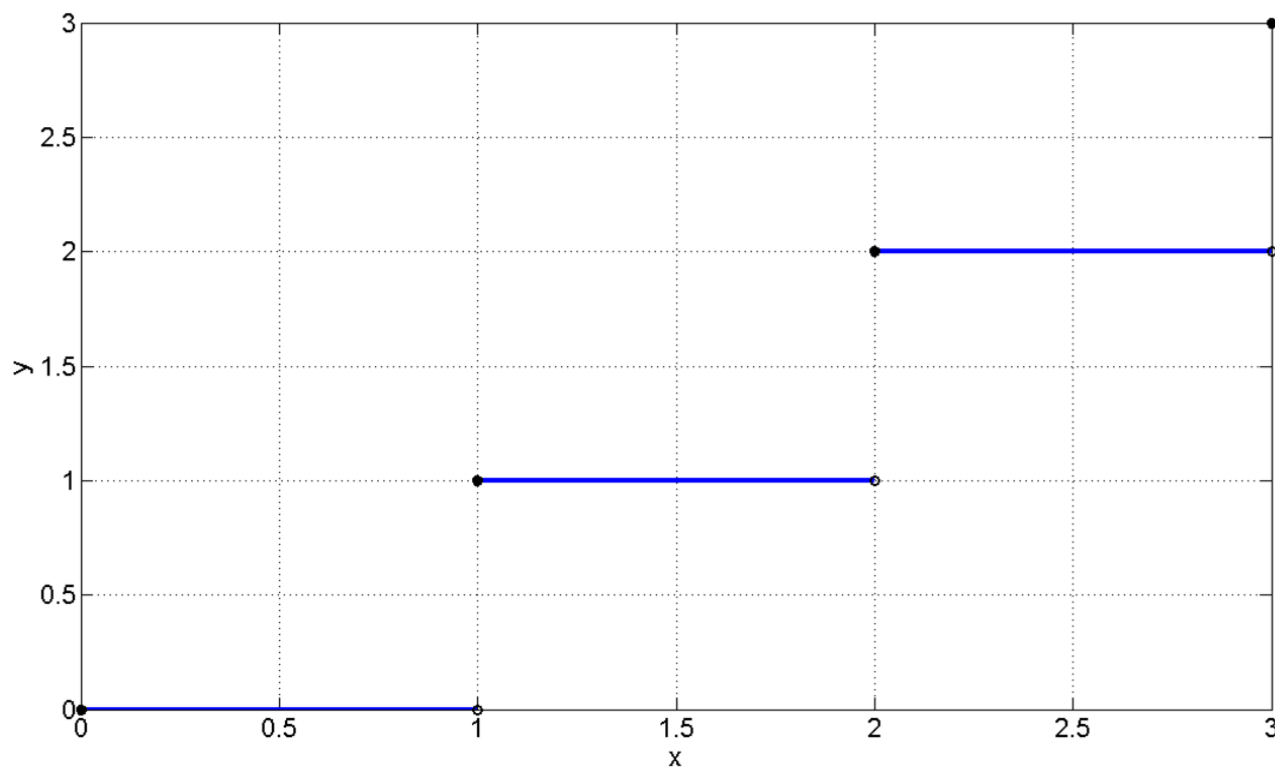

Figure 2: *A plot of the function $g(x) = \lfloor x \rfloor$.*

We have the following:

(a) $\lim\limits_{x \to 2^-} \lfloor x \rfloor = 1$

(b) $\lim\limits_{x \to 2^+} \lfloor x \rfloor = 2$

(c) $\lim\limits_{x \to 2}$ does not exist

The last limit does not exist because the right-hand limit and the left-hand limit are different. If we just plug 2 into the function we obtain $\lfloor 2 \rfloor = 2$, which is NOT the same as $\lim\limits_{x \to 2} \lfloor x \rfloor$ since the limit does not even exist. So, how do we know if just "plugging in" works for a given function?

The answer will come in section 1.6 when we study continuous functions. If the function has any jumps (like $\lfloor x \rfloor$ at $x = 2$) or is going crazy at any point (like $\sin(\frac{1}{x})$ is at 0), then we cannot simply plug in to find the limit, as we saw in class.

The reason I wanted to send you this is that I believe I misled you by saying if you just plug in for the limit and obtain a number you are done. This is true in many cases but not in others (such as in the floor function example above). So, please be careful when computing limits. One way to check your answer is to graph the function. Plugging in the number is a great first step that works often; however, it is a good idea to convince yourself that you have the correct answer by graphing the function as well.

Finally, I wanted to let you know that there is a method for computing limits that works all of the time, although it can get complicated (it is discussed in section 1.2).

If you have any questions please do not hesitate to ask. I want to make sure you understand limits well as they are very important in calculus. Chapter 1 in the text focuses on limits, so we will spend the next couple of weeks on them. There will be many cases when "just plugging in" works, but we will also learn how to distinguish many cases in which "just plugging in" does not work or when "just plugging in" gives the incorrect answer (like for the floor function). Again, I apologize for any confusion I caused.