QUIZZES AND EXAMS FOR MATH 1210
CALCULUS I

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Week 1 Quiz

Problem 1. Sketch the graph of the following function

$$f(x) := \begin{cases} \sqrt{x} & \text{if } 0 \leq x < 1 \\ \lfloor x \rfloor & \text{if } 1 \leq x \leq 3 \\ x^2 & \text{if } x > 3 \end{cases} .$$

Then find each of the following, or state that does not exist:

$$\lim_{x \to 1^-} f(x) = \dots$$
$$\lim_{x \to 1^+} f(x) = \dots$$
$$\lim_{x \to 1} f(x) = \dots$$
$$f(1) = \dots$$

$$\lim_{x \to 2^-} f(x) = \dots$$
$$\lim_{x \to 2^+} f(x) = \dots$$
$$\lim_{x \to 2} f(x) = \dots$$
$$f(2) = \dots$$

$$\lim_{x \to 3^-} f(x) = \dots$$
$$\lim_{x \to 3^+} f(x) = \dots$$
$$\lim_{x \to 3} f(x) = \dots$$
$$f(3) = \dots .$$

Problem 2. Suppose that $\lim_{x \to a} f(x) = 27$ and $\lim_{x \to a} g(x) = -2$. Find the following limit

$$\lim_{x \to a} \frac{\sqrt[3]{f(x)}}{|g^2(x) + 3g(x)|}.$$

Date: Fall 2014.

Week 2 Quiz

Problem 1. Find the following limit

$$\lim_{x\to 4^+} \frac{x-4}{\sqrt{x^2-3x-4}}.$$

Problem 2. Find the following limit

$$\lim_{x\to 0} \frac{\tan(2x)}{\sin(3x)\cdot\cos(x)}.$$

Week 3 Quiz

Problem 1. Find the following limit

$$\lim_{x\to\infty} \frac{\sin(x)\cos(x)}{x^2}.$$

Problem 2. Find the following limit

$$\lim_{x\to\infty} (\sqrt{x^2+3x}-x).$$

Week 4 Super Quiz

Problem 1. Find the following limit

$$\lim_{x\to\infty} \sqrt[3]{\frac{x^2+3x-1}{x(8x-1)}}.$$

Problem 2. Find the following limit

$$\lim_{x\to 2^+} \frac{\sqrt{x^2-x-2}}{x^2-5x+6}.$$

Problem 3. Find all points on the graph $y=\sin(x)\cos(x)$ where the tangent line is horizontal.

Problem 4. Find the equation of the tangent line to $y=\sin(x)\cos(x)$ at the point $(\frac{\pi}{6},\frac{\sqrt{3}}{4})$.

Week 5 Quiz

Problem 1. Let $f(x):=\cos(3x)$. Find f', f'', $f^{(3)}$, $f^{(4)}$, and $f^{(2014)}$.

Problem 2. Find the following limit

$$\lim_{x\to\infty} \cos(x^2) \sin\left(\frac{1}{x^2-1}\right).$$

Midterm 1

Problem 1. Consider the following function

$$f(x) = \begin{cases} \frac{1}{(x-1)^2} & \text{if} \quad 0 \le x < 1 \quad \text{or} \quad 1 < x \le 2 \\ \lfloor x \rfloor & \text{if} \quad 2 < x \le 4 \\ \frac{-5}{x^2-4x-5} & \text{if} \quad 4 < x < 5 \quad \text{or} \quad x > 5 \end{cases}.$$

a) Sketch the graph of $f(x)$.
b) Find the vertical and horizontal asymptotes of the graph of $f(x)$.
c) Find each of the following, or state that does not exist:

$$\lim_{x\to 1^-} f(x) = \dots \qquad \lim_{x\to 1^+} f(x) = \dots \qquad \lim_{x\to 1} f(x) = \dots \qquad f(1) = \dots$$

$$\lim_{x\to 2^-} f(x) = \dots \qquad \lim_{x\to 2^+} f(x) = \dots \qquad \lim_{x\to 2} f(x) = \dots \qquad f(2) = \dots$$

$$\lim_{x\to 3^-} f(x) = \dots \qquad \lim_{x\to 3^+} f(x) = \dots \qquad \lim_{x\to 3} f(x) = \dots \qquad f(3) = \dots$$

$$\lim_{x\to 4^-} f(x) = \dots \qquad \lim_{x\to 4^+} f(x) = \dots \qquad \lim_{x\to 4} f(x) = \dots \qquad f(4) = \dots$$

$$\lim_{x\to 5^-} f(x) = \dots \qquad \lim_{x\to 5^+} f(x) = \dots \qquad \lim_{x\to 5} f(x) = \dots \qquad f(5) = \dots.$$

Problem 2. a) Find the following limit

$$\lim_{x\to\infty} \cos\left(\frac{(1-\pi x)(x+\pi)}{(x+6)(1-6x)}\right).$$

b) Find the following limit

$$\lim_{x\to\infty} \cos(x^2) \cdot \cos^2(x) \cdot \sin\left(\frac{x^2+1}{x^2(1-2x)}\right).$$

Problem 3. Let

$$f(x) = \frac{\sin(x-3)}{\sqrt{x^2-2x-3}}.$$

a) Find the domain of the function $f(x)$.
b) Find the limit

$$\lim_{x\to 3^+} f(x).$$

Problem 4. a) Find the equation of the tangent line to $y = 3 + x\sin(3x)$ at $\left(\frac{\pi}{3}, 3\right)$.
b) Find the derivative of

$$\sqrt[3]{\sin\left(\cos\left(\frac{3}{x}\right)\right)}.$$

Week 9 Quiz

Problem 1. i) Find the critical points of the following function

$$f(x) = x^2(x^2 - 8).$$

ii) Use the first derivative test to decide which critical points give a local maximum value and which give a local minimum value.
iii) Use the second derivative test to solve the previous problem.

Problem 2. Given

$$f'(x) = -(x-1)(x-3)^2(x-4)(x-5),$$

find all values of x that make the function $f(x)$ a local maximum, and all values that make $f(x)$ a local minimum.

Week 10 Quiz

Problem. Consider the following function

$$f(x) = \frac{4 - x^2}{x^2 - 1}.$$

i) What is the domain of $f(x)$? What are the x-intercepts and the y-intercept? Is the function even, odd, or neither of the two?
ii) Find the critical points.
iii) When is $f(x)$ increasing? When is $f(x)$ decreasing? Find local maxima and minima.
iv) When is $f(x)$ concave upward? When is $f(x)$ concave downward?
v) Find the asymptotes of $f(x)$.
vi) Sketch the graph of $f(x)$.

Week 11 Super Quiz

Problem 1. Decide whether the Mean Value Theorem applies to the following two functions on the interval $I = [-8, 27]$. If it does, find all possible values of c; if not,

state the reason:

$$f(x) = x^{\frac{2}{3}}, \qquad g(x) = x^{\frac{4}{3}}.$$

Problem 2. Find the general antiderivative of the following function

$$f(x) = x^5 - 4x^3 - \frac{1}{2}x^2 - 14\sqrt{x}.$$

Problem 3. Find the critical points of the following function

$$f(x) = x(x^2 + 3x - 9).$$

Use the test of your choice to decide which of the critical points give a local maximum and which give a local minimum.

Problem 4. Given

$$f''(x) = (2 - x)(x - 4)(x^2 + 4)(x + 3),$$

find all inflection points of $f(x)$. Determine where $f(x)$ is concave up and where it is concave down.

Midterm 2

Problem 1. (*6+6 points*) Evaluate the following integrals:

$$a) \quad \int \frac{3\sqrt{z} + 2\sin(z)}{(\cos(z) - \sqrt{z^3})^2} dz, \qquad b) \quad \int \frac{(3x-1)^2}{\sqrt[3]{x}} dx.$$

Problem 2. a) (*6 points*) Solve the following differential equation

$$\frac{dy}{dx} = \frac{x^2(1 + x^3)}{\sqrt{y}}$$

subject to the condition $y = 9$ at $x = -\sqrt[3]{2}$.
b) (*6 points*) Solve the following differential equation

$$\frac{dy}{dx} = y^2 \cos(x)$$

subject to the condition $y = 0$ at $x = \frac{\pi}{3}$.

Problem 3. (*6 points*) Decide whether the Mean Value Theorem applies to the following two functions on the interval $I = [0, 33]$. If it does, find all possible values of c; if not, state the reason:

$$f(x) = (x - 1)^{\frac{3}{5}}, \qquad g(x) = (x - 1)^{\frac{6}{5}}.$$

Problem 4. (*10 points*) Consider the following function

$$f(x) = \frac{(x-1)^3}{8-x^3}.$$

a) Find the domain. Is the function even, odd, or neither of the two?
b) Find the vertical and horizontal asymptotes.
c) Find the critical points.
d) Determine where the function is increasing, where it is decreasing, and find the local extreme values.

Week 15 Quiz

Problem 1. Find the area of the plane region bounded by $y = \cos(x)$, $y = \sin(x)$, the y-axis, and the line $x = \pi/4$.

Problem 2. The plane region below $y = x - x^2$ and above the x-axis is revolved about the y-axis. Compute the volume of the resulting solid.

Final Exam

Problem 1. Consider the function

$$f(x) = \frac{x}{x^2-1}.$$

a) (*3 points*) Find the domain, the x-intercepts, and the y-intercepts. Is the function even, odd, or neither of the two?
b) (*2 points*) Find the horizontal and vertical asymptotes.
c) (*2 points*) Find the critical points. When is $f(x)$ increasing? When is $f(x)$ decreasing?
d) (*2 points*) Find the inflection points. When is $f(x)$ concave upward? When is $f(x)$ concave downward?
e) (*2 points*) Sketch the graph of $f(x)$.

Problem 2. a) (*4 points*) Find the following limit

$$\lim_{x\to\infty} \left(\frac{(1-x)(2\pi x - 1)}{\pi x(2-x)} \right)^3.$$

b) (*5 points*) Solve the following differential equation

$$\frac{dy}{dx} = \frac{x^2}{y^2(1+x^3)^2}$$

subject to the condition $y = 2$ at $x = 0$.

Problem 3. a) (*3 points*) Find $G'(x)$, where

$$G(x) := \frac{1}{x}\int_0^{-x} f(z)dz.$$

b) (*4 points*) Evaluate the following definite integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{\sin(\theta)}{\sqrt{\cos(\theta)}}d\theta.$$

c) (*4 points*) Evaluate the following indefinite integral

$$\int \frac{(1-x^2)^2}{\sqrt{x^3}}dx.$$

Problem 4. (*5 points*) Compute the area of the region between $x = y^4$ and $x = 1$.

Problem 5. Consider the plane region R below the graph of the function $y = x^2 - x^3$ and above the x-axis, for $x \geq 0$.

a) (*5 points*) Compute the volume of the solid obtained by revolving R about the x-axis.

b) (*5 points*) Compute the volume of the solid obtained by revolving R about the y-axis.

c) (*4 points*) Set up an integral for the volume of the solid obtained by revolving R about the line $y = -1$ (you don't have to evaluate the integral in this case).

Problem 6. a) (*5 points*) Find the length of the curve given by

$$\begin{cases} x = t^6 + 36 \\ y = \frac{3}{2}(1 - t^4) \end{cases}$$

for $t \in [0, 1]$.

b) (*5 points*) Find the area of the surface of revolution generated by revolving the curve $y = \sqrt{x-1}$, for $x \in [2, 4]$, about the x-axis.