Humboldt Universität zu Berlin October 19, 2009
Algebraic Geometry I
Lectures by Prof. Dr. R. N. Kloosterman
Exercises by N. Tarasca

Exercise Sheet 1

Hand in solutions not later than Monday, October 26.

Exercise 1 Let k be a field. Determine whether the following subsets of $M_n(k)$ are algebraic or not:

a) the set of symmetric matrices;

b) for $k = \mathbb{C}$ the set of Hermitian matrices.

Exercise 2 Let $k = \mathbb{R}$ and $q(x_1, x_2, x_3) = x_1^2 + x_2^2 - x_3^2$.

a) Determine whether $X := \{x \in \mathbb{R}^3 \mid q(x) = -1 \,\text{and}\, x_3 > 0\}$ is algebraic or not in $\mathbb{R}^3$.

b) Determine whether $O(2,1) := \{g \in GL_3(\mathbb{R}) \mid q(g(x)) = q(x) \,\forall x \in \mathbb{R}^3\}$ is algebraic or not in $GL_3(\mathbb{R})$.

Exercise 3 Let $R = \mathbb{R}[x, y, z]$.

a) Let

$$I_1 = (x^2+y^2-1, z), \quad I_2 = (x^2+y^2+z^2-1, z), \quad I_3 = (x^2+y^2-1, z-1)$$

be ideals of R. Determine whether $I_i = I_j$ for $i \neq j$ and sketch $V(I_i)$ for $i, j = 1, 2, 3$.

b) Let

$$J_1 = (x^2 + y^2 + z^2 - 1, x^2 + y^2 - z^2 + 1), \quad J_2 = (z^2 - 1, x^2, x^2 + y^2),$$
$$J_3 = (x^2 + y^2, z^2 - 1)$$

be ideals of R. Determine whether $J_i = J_j$ for $i \neq j$ and sketch $V(J_i)$ for $i, j = 1, 2, 3$.

Exercise 4 Determine whether the following rings are Noetherian or not:

a) $\mathbb{Z}[\sqrt{5}]$

b) $\mathbb{Z}[\sqrt{5}, \pi]$.