

* REVIEW *

Chapter 1: Ways to solve 1st order ODE

- 1) Integrate $y' = f(x)$
- 2) separation $y' = f(x)g(y)$
- 3) integrating factor $y' + P(x)y = Q(x)$
 $y(x) = \exp(\int P(x) dx)$

application:

- Toricelli's law
 $A(y) \frac{dy}{dt} = -k\sqrt{y}$ where $k = a\sqrt{2g}$
- Mixture problem
 $\frac{dx}{dt} = c_1 r_1 - r_0 \frac{x}{V}$

Chapter 2:

- Equilibrium soln & stability - phase line diagram
- application: -- population model
 - exponential growth $p' = kp$
 - logistic growth $p' = k(M-p)p$
- acceleration/velocity: $mv' = -mg + F_R$

Chapter 3:

- Linear eqn with constant coeffs ~ multiple roots?
 ~ characteristic eqn
- Particular & Complementary soln for non-homogeneous eqn
- Mass & spring: $mx'' + cx' + kx = f(t)$
 - undamped ($c=0$)
 natural frequency $\omega_0 = \sqrt{\frac{k}{m}}$
 resonance when forced ω / natural frequency
 - damped case ($c \neq 0$)
 - overdamped case: 2 real roots ($c^2 > 4km$)
 - critically damped: 1 real roots ($c^2 = 4km$)
 - underdamped case: 2 complex roots ($c^2 < 4km$)
- also linear independence using Wronskian

Chapter 5: Systems of ODEs

- Eigenvalue method
- Fundamental matrix solution λ e^{At}
 Generalized eigenvector

application:

- mass & spring system
- mixture problem

Chapter 6: Phase plane analysis

- Linear analysis of equilibrium soln (Jacobian)

Chapter 7: Laplace transform

- How to solve ODE using LT
- Properties: Convolution

Translation $\mathcal{L}\{H(t-a)f(t-a)\} = e^{-as}F(s)$

$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

Partial fraction

Chapter 9: Fourier Series

- definition of FS
- odd extension, Fourier sine series
- even extension, Fourier cosine series
- PDE: separation of variables.