Math 2280 - Maple Assignment 3

Earthquake Project

Due Wednesday, November 1, 2006

Reference: Edwards and Penny (Section 5.3, p. 326-32).
You will need to use Maple version 9 or higher for this assignment. If you are working from a computer lab in the Math department, type **xmapleV9** or **xmapleV10** from the terminal.

In this project, you will investigate the effects of an earthquake on a multi-stories building using a model described in your text book. The idea is to model the building as a mass and spring system with the spring providing a horizontal restoring force to any displacement. The first floor is connected to the ground by a spring and the second floor is connected to the first floor by another spring and so on. The model we will use here is identical to the one in the text except that we take a building with 6 stories.

Problems

1. Let the mass of each story be $m = 1000$ slugs and the spring constant be $k = 10000$ lb/ft. Define the 6x6 mass matrix M and stiffness matrix K. Note that $k_7 = 0$ as the top floor is not tethered at the top. Convert the system $\boldsymbol{M}\boldsymbol{x}'' = \boldsymbol{K}\boldsymbol{x}$ to $\boldsymbol{x}'' = \boldsymbol{A}\boldsymbol{x}$.

2. **Eigenvalues and Natural Frequencies**
   Find the eigenvalues λ_i of the matrix A above. What are the natural frequencies $\omega_i = \sqrt{-\lambda_i}$ and the natural frequencies of vibrations?

3. **Earthquake**
   Now model an earthquake by introducing an external periodic forcing term to the system. Let $\boldsymbol{b}$ be the vector $[1\ 1\ 1\ 1\ 1\ 1]^T$ and take the external force vector per unit mass to be $\boldsymbol{f}(t) = \omega^2 \cos(\omega t)\boldsymbol{b}$.
   The solution to the system
   $$\boldsymbol{x}'' = \boldsymbol{A}\boldsymbol{x} + \omega^2 cos(\omega t)\boldsymbol{b}$$
   is the sum of the complementary solution and the particular system, $\boldsymbol{x}(t) = \boldsymbol{x}_c(t) + \boldsymbol{x}_p(t)$. In our current model, we do not include any frictional damping term. Physically however, every mechanical system has some fricitional resistance, no matter how small. Typically, the result of including the frictional term is that the complementary solution $\boldsymbol{x}_c$ will be damped out,
   $$\boldsymbol{x}_c(t) \to \boldsymbol{0} \text{ as } t \to \infty$$
   Thus, the long term behavior of the system is determined by the particular solution $\boldsymbol{x}_p(t)$.
   $$\boldsymbol{x}(t) \to \boldsymbol{x}_p(t) \text{ as } t \to \infty$$
   As we discussed in class, the particular solution will be $\boldsymbol{x}_p(t) = \boldsymbol{c}\cos(\omega t)$ where $\boldsymbol{c}$ satisfies $(\boldsymbol{A} + \omega^2 \boldsymbol{I})\boldsymbol{c} = -\omega^2 \boldsymbol{b}$. The components of the vector $\boldsymbol{c}$ give the motion of each floor of the

building for this problem.

Define a function $C(T)$ that gives the maximum amplitude of of vibration of vibration of the floor of the building for a given external force with period $T = 2\pi/\omega$. Plot this function over a reasonable range of T. To see the plot clearly, you will need to also define the y-axis over a range that is not too big. You should see several spikes on your plot. Explain your plot. What do the values of the spikes correspond to?

4. Eartquake Damage
Use the function $C(T)$ to assess possible damages due to an earthquake. For the first spike, graph the function $C(T)$ over a small enough interval around the spike to determine an approximate interval within which some floor of the building will undergo oscillations in excess of five feet from equilibrium.

Maple Help

To do matrix vector operation in Maple, load the linalg library,

```
> with(linalg):
```

You can define matrices and vector using the command Matrix. For example, use the following commands to define

$$\boldsymbol{P} = \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

```
> P:= Matrix([[2,4],[3,7]]);
> v := Matrix([[1],[1],[1]]);
```

The format is such that you need to define each row inside [ ]. You can also use input a diagonal matrix and a banded matrix using some shortcut,

$$\boldsymbol{M} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \boldsymbol{Q} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 3 & 2 \end{bmatrix}$$

```
> M:= 10*Matrix(3,3,shape=identity);
> B := Matrix([[3,3,3],[2,2,2,2],[1,1,1]], shape=band[1,1], scan=band[1,1]);
```

To find the inverse of a matrix M, use the inverse command,

```
> Minv := Matrix(inverse(M));
```

To do a matrix/matrix or matrix/vector multiplication, use '.',

```
> AB := A.B;
```

A and B will have to be defined using the Matrix command initially.
To find eigenvalues and eigenvectors,

```
> lambda := eigenvals(A);
```

For problem 2, use the command eigenvals as above to find the eigenvalues of the matrix $\boldsymbol{A}$. To find the frequencies and periods easier, create a vector with entries equal to the eigenvalues, then define a function to give the frequency (or period) in terms of the eigenvalues. Use the map command to evaluate the function for all eigenvalues.

```
> lambda := [eigenvals(A)];
> freq:= v -> sqrt(-v);
> omega0:= map(freq,lambda);
```

Do something similar to get Maple to compute the period.

For problem 3, first define the vector $\boldsymbol{b}$ and the matrix $\boldsymbol{B} = \boldsymbol{A} + \omega^2 \boldsymbol{I}$ using the Matrix command as discussed before. Then compute c by multiplying $\boldsymbol{B}^{-1}$ to $-\omega^2 \boldsymbol{b}$,

```
> c:= Matrix(Matrix(inverse(B)).(-omega^2*b));
```

Find the maximal element by using the command norm in Maple,

```
> C:= t -> subs(omega=2*Pi/t,norm(c,infinity));
```

Now $C(t)$ is a Maple function and you can plot it as you normally would plot a function.