

Math 2280 - Fall 2006
Sample code for Lab 1

Part (1)

Input the function A(t) as a function in Maple

```
> A := t -> 34 - 10*cos(omega*(t-3));
```

$$A := t \rightarrow 34 - 10 \cos(\omega(t-3))$$

Type your differential equation into Maple

```
> eq1 := diff(u(t), t) = k*(A(t) - u(t));
```

$$eq1 := \frac{d}{dt} u(t) = k(34 - 10 \cos(\omega(t-3)) - u(t))$$

Now, use Maple to solve this equation using initial condition u(0)=u0

```
> dsolve({eq1, u(0)=u0}, u(t));
```

$$u(t) = \frac{e^{(-kt)} (-34 k^2 - 34 \omega^2 + 10 k^2 \cos(3 \omega) - 10 k \omega \sin(3 \omega) + u0 k^2 + u0 \omega^2)}{k^2 + \omega^2} - \frac{2 (-17 k^2 - 17 \omega^2 + 5 k^2 \cos(\omega t - 3 \omega) + 5 k \omega \sin(\omega t - 3 \omega))}{k^2 + \omega^2}$$

There! Maple just solved this equation for you. Now, to make life easier so you can plot u(t) easily later on, define the solution as a Maple function by typing

```
> u := unapply(rhs(dsolve({eq1, u(0)=u0}, u(t))), t);
```

$$u := t \rightarrow \frac{e^{(-kt)} (-34 k^2 - 34 \omega^2 + 10 k^2 \cos(3 \omega) - 10 k \omega \sin(3 \omega) + u0 k^2 + u0 \omega^2)}{k^2 + \omega^2} - \frac{2 (-17 k^2 - 17 \omega^2 + 5 k^2 \cos(\omega t - 3 \omega) + 5 k \omega \sin(\omega t - 3 \omega))}{k^2 + \omega^2}$$

Part (2)

Which part of the above is the steady state part i.e. the part that persist when you take $t \rightarrow \infty$

Part(3)

Type in parameter values into Maple then plot the functions

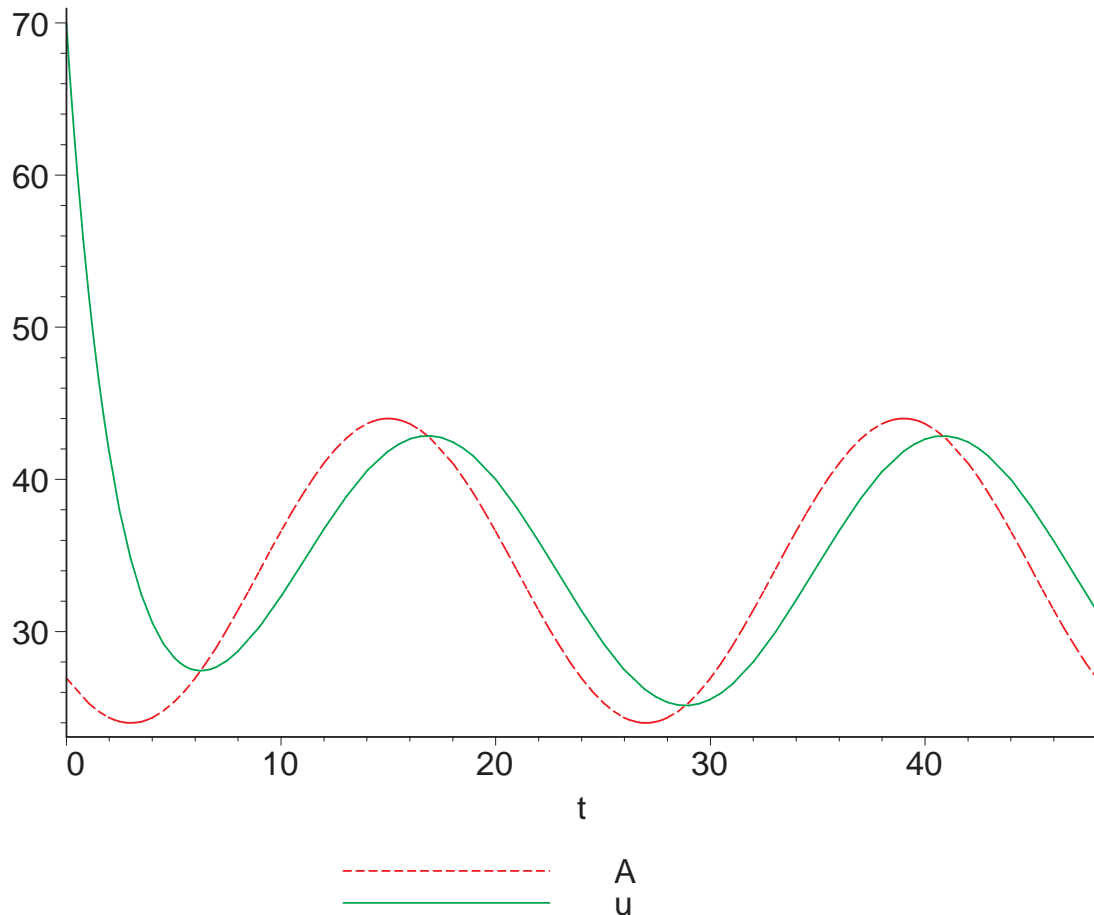
```
> omega := Pi/12; k := 0.5; u0 := 70;
```

$$\omega := \frac{\pi}{12}$$

$$k := 0.5$$

$$u0 := 70$$

```
> plot([A(t), u(t)], t=0..48, linestyle=[DASH, SOLID], legend=['A', 'u']);
```



You just plotted the two functions with t going from 0 to 48 (2 days).

Now, try to estimate the phase delay from your graph. Note how the external temperature $A(t)$ is leading the internal temperature $u(t)$ as $A(t)$ reaches its peak first. The phase delay is then just the distance between the peak of $A(t)$ and $u(t)$.

You also need to find the phase delay analytically using the trig relationship described in the problem sheet.

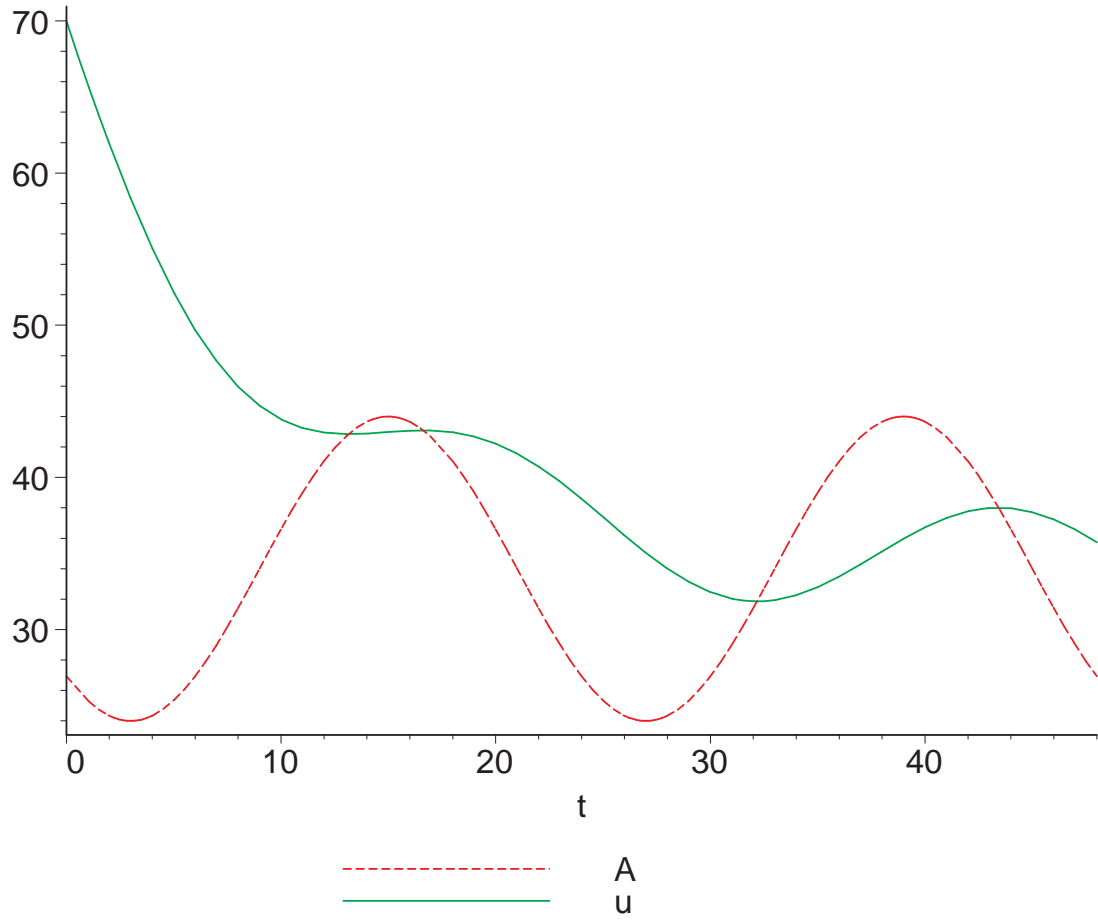
Part (4)

Now, use different parameter values.

```
> k:= 0.1;
```

```
      k:= 0.1
```

```
> plot([A(t),u(t)],t=0..48,linestyle=[DASH,SOLID],legend=['A','u']);
```



Do the same thing with $k=0.3$. Compare how your solution changes as you change $k = 0.1, 0.3, 0.5$
 Answer the questions listed in your problem sheet.