Review of Chapter 4 and 5 for Exam 2

Math 2250-2 Summer 2007

Exam 2 covers materials from sections 4.1-4.4 and 5.1-5.5 of the text.

Chapter 4
Important definitions to know:

(a) **Vector Space**: A collection of objects which can be added and scalar multiplied, so that the usual arithemetic properties (page 240) hold. You do not need to memorize all eight of these properties.
The key point is that not only is $\mathbf{R^n}$ a vector space, but also certain subsets of it are, and so are spaces made out of functions because functions can be added and scalar multiplied (page 265).

(b) **Subspace**: a subset of a vector space which is itself of vector space.
To check whether a subset is actually a subspace you only have to show that sums and scalar multiples of subset elements are also in the subset (Theorem 1 page 242).
Examples of important subspaces are the set of homogeneous solutions to a matrix equation $\mathbf{Ax{=}0}$ (page 243), the span of a collection of vectors (page 248), and the set of homogenous solutions to a linear differential equation (section 5.2).

(c) A **linear combination** of a set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}\}$ is any expression $c_1\mathbf{v_1} + c_2\mathbf{v_2} + ... + c_n\mathbf{v_n}$ for some scalar coefficients $c_1, c_2, ..., c_n$ (page 246).

(d) The **span** of a set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}\}$ is the collection of all linear combinations (page 248).

(e) A collection $\{\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}\}$ is **linearly independent** if and only if the only solution to $c_1\mathbf{v_1} + c_2\mathbf{v_2} + ... + c_n\mathbf{v_n} = \mathbf{0}$ is $c_1 = c_2 = ... = c_n = 0$.
If there is a set $c_1, c_2, ..., c_n$ not *all* zeros such that solves the equation then the vectors are **linearly dependent** (page 249).

(f) A **basis** for a vector space (or subspace) is a set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}\}$ which *span* the space and which are *linearly independent* (page 255).

(g) The **dimension** of a vector space is the number of elements in any basis.

Useful Facts:

(a) If the dimension of a vector space is n, then no collection of fewer than n vectors can span and every collection with more than n elements is dependent.
For example, you need *at least* n vectors to span $\mathbf{R^n}$. A set of n linearly independent vectors spans $\mathbf{R^n}$.

(b) n vectors in $\mathbf{R^n}$ forms a basis if and only if the square matrix that results from putting the vectors as columns is non-singular.
So you can use the determinant or RREF as a way to test for span and/or linear independence.

Make sure you know how to do these computations:

(a) Be able to check whether vectors are independent or independent, e.g. problems on page 248 in 4.3
Know how to use RREF to check for linear independence and write down dependencies if the vectors are not linearly independent.

(b) Be able to find bases for the solution space to homogeneous equations $\mathbf{Ax} = \mathbf{0}$, e.g. problems on page 255 in 4.4.

Chapter 5

(a) Two functions are linearly independent if they are not constant multiples of each other. One way to check for linear independence is to use the Wronskian. Make sure you know how to check linear independence given n functions, $\{f_1(x), f_2(x), f_3(x), ..., f_n(x)\}$, by showing that their Wronskian is not equal to 0.

(b) Homogeneous linear n-th order equation

$$p_n(x)y^{(n)} + p_{n-1}(x)y^{(n-1)} + ... + p_1(x)y' + p_0(x)y(x) = 0$$

General solution consists of n linearly-independent solution. That is, all possible solutions to this equation can be writeen as a linear combination of n functions (i.e. the solution space is an n dimensional vector space). Be able to solve for a specific solution when initial conditions are given.

(c) Homogeneous equation with constant coefficients

$$a_n y^{(n)} + a_{n-1}y^{(n-1)} + ... + a_1 y' + a_0 y(x) = 0$$

Know how to find a general solution consisting of exponentials using characteristic equations. What to do when the root is complex? What about when you have repeated roots?

(d) Non-homogeneous linear equation

$$p_n(x)y^{(n)} + p_{n-1}(x)y^{(n-1)} + ... + p_1(x)y' + p_0(x)y(x) = f(x)$$

The general solution consists of a complemetary solution (which solves the corresponding homogeneous problem) and a particular solution.
Be able to solve a non-homogeneous equation for equations with constant coefficients using methods of undetermined coefficients. Know what to do when $f(x)$ is not linearly independent to the complementary solution.

(e) Application: mechanical vibrations
Only unforced system will be tested in Exam 2

$$mx'' + cx' + kx = 0$$

Make sure you are able to derive this equation from Newtons and Hookes Laws.
Know what the solution is and what it means for the different cases.

- undamped motion ($c = 0$):
  Be able to go from $A\cos(\omega t) + B\sin(\omega t)$ to $C\cos(\omega t - \alpha)$ using the ABC triangle to find the amplitude C and phase α.
- damped case ($c \neq 0$):
  There are three possible cases: under-damped, over-damped, critically damped. Know how to recognize, and different forms of the solution.