Math 2250-2 Maple Assignment 3

Earthquake Project

Due Friday, August 3, 2007

Reference: Edwards and Penny (Section 7.4, p. 440-441).

You will need to use Maple version 9 or higher for this assignment. If you are working from a computer lab in the Math department, type **xmapleV9 &** or **xmapleV10 &** from the terminal.

In this project, you will investigate the effects of an earthquake on a multi-stories building using a model described in your text book. The idea is to model the building as a mass and spring system with the spring providing a horizontal restoring force to any displacement. The first floor is connected to the ground by a spring and the second floor is connected to the first floor by another spring and so on. The model we will use here is identical to the one in the text except that we take a building with 6 stories.

Modify the given Maple worksheet by defining parameter values and some functions appropriately.

Problems

1. Let the mass of each story be $m := 1000$ slugs and the spring constant be $k := 10000$ lb/ft. Define the 6x6 mass matrix M and stiffness matrix K. Note that $k_7 = 0$ as the top floor is not tethered at the top. Convert the system $\boldsymbol{M}\boldsymbol{x}'' = \boldsymbol{K}\boldsymbol{x}$ to $\boldsymbol{x}'' = \boldsymbol{A}\boldsymbol{x}$.

2. **Eigenvalues and Natural Frequencies**
   Find the eigenvalues λ_i of the matrix A above. What are the natural frequencies $\omega_i = \sqrt{-\lambda_i}$ and the natural frequencies of vibrations?

3. **Earthquake**
   Now model an earthquake by introducing an external periodic forcing term to the system. Let $\boldsymbol{b}$ be the vector $[1\ 1\ 1\ 1\ 1\ 1]^T$ and take the external force vector per unit mass to be $\boldsymbol{f}(t) = \omega^2 \cos(\omega t)\boldsymbol{b}$.

   The solution to the system

   $$\boldsymbol{x}'' = \boldsymbol{A}\boldsymbol{x} + \omega^2 cos(\omega t)\boldsymbol{b}$$

   is the sum of the complementary solution and the particular system, $\boldsymbol{x}(t) = \boldsymbol{x}_c(t) + \boldsymbol{x}_p(t)$. In our current model, we do not include any frictional damping term. Physically however, every mechanical system has some fricitional resistance, no matter how small. Typically, the result of including the frictional term is that the complementary solution $\boldsymbol{x}_c$ will be damped out,

$$\boldsymbol{x}_c(t) \to \mathbf{0} \text{ as } t \to \infty$$

Thus, the long term behavior of the system is determined by the particular solution $\boldsymbol{x}_p(t)$.

$$\boldsymbol{x}(t) \to \boldsymbol{x}_p(t) \text{ as } t \to \infty$$

As we discussed in class, the particular solution will be $\boldsymbol{x}_p(t) = \boldsymbol{c}\cos(\omega t)$ where $\boldsymbol{c}$ satisfies $(\boldsymbol{A} + \omega^2 \boldsymbol{I})\boldsymbol{c} = -\omega^2 \boldsymbol{b}$. The components of the vector $\boldsymbol{c}$ give the motion of each floor of the building for this problem.

Define a function $C(\omega)$ that gives the maximum amplitude of of vibration of vibration of the floor of the building for a given external force with frequency ω. Plot this function over a reasonable range of ω. To see the plot clearly, you will need to also define the y-axis over a range that is not too big. You should see several spikes on your plot. Explain your plot. What do the values of the spikes correspond to?

4. Eartquake Damage
   Use the function $C(\omega)$ to assess possible damages due to an earthquake. For the first spike, graph the function $C(\omega)$ over a small enough interval around the spike to determine an approximate interval within which some floor of the building will undergo oscillations in excess of five feet from equilibrium.