# Math 2250-2 Maple Assignment 2

Forced Oscillations and Resonance

Due Wednesday, July 18, 2007

**Reference:** Edwards and Penny (Section 5.6)

The goal of this project is to study solutions to the second order equation describing the motion of a mass-spring-dashpot system and explore the phenomenon of resonance.

Recall that the basic equation describing the motion of a mass attached to a spring and a dashpot is

$$mx''(t) + cx'(t) + kx = F(t)$$

where $m$ is the mass, $k$ is the spring constant, and $c$ is the damping constant of the dashpot. The motion is called *undamped* if $c = 0$ and *damped* if $c > 0$. We refer to the motion as *free* if $F(t) = 0$ and *forced* if $F(t) \neq 0$.

## 1 Undamped Forced Oscillations

Consider the following initial value problem which describes a forced undamped motion,

$$x'' + 2.5x = 7\cos(\omega t)$$

$$x(0) = 0 \text{ and } x'(0) = 0$$

(a) What is the natural frequency of the system?

(b) Set $\omega$ to equal five times the natural frequency. Solve the resulting initial value problem by using **dsolve**. Then, plot the solution on a suitable interval to show the global behavior of the solution.

(c) The solution to this equation is the sum of the particular solution and the complementary solution. In this case, the two functions have different periods. Find the exact period of the solution (see Example 1 on p. 350 in the text).

## 2 Practical Resonance

Consider the damped forced motion described by the following initial value problem,

$$x'' + cx' + 25x = 7\cos(\omega t)$$

$$x(0) = 0 \text{ and } x'(0) = 0$$

(a) Consider each value of the damping constant, $c = 1/2$, $c = 1$, and $c = 2$. Compute the amplitude function $C(\omega)$ for these values of $c$ and plot the three functions on one graph. Make sure you label the plot and indicate which curve corresponds to which value of $c$.
The function $C(\omega)$ is given on page 355, equation (21). We also derive this function in class.

(b) For each of the three values of $c$, find the value of $\omega$ at which practical resonance occurs. What are the values of $C(\omega)$ at each of these points? How is the amplitude changing as $c$ becomes smaller? Explain why this is the case.

## 3 Resonance

(a) Consider the system given in problem 1 again, but now replace $\omega$ by the natural frequency of the system. Solve the resulting system again and graph the new solution. Explain what is the difference between the two solutions. In particular, note how the amplitudes of the motions are changing with time $t$.

(b) Solve the system given in problem 2. Take $c = 1/2$ and $\omega$ to be the frequency at which practical resonance occurs. Plot your solution. What is the difference between the practical resonance observed here and the resonance for the undamped case in (a) above.

## Notes

(a) You might want to **restart** before beginning each separate problem on this assignment.

(b) Remember how to solve an ODE using **dsolve**. First input the equation,

```
>  eq:= m*diff(x(t),t,t) + c*diff(x(t),t) + k*x(t) = F(t);
```

To solve this equation using the initial condition $x(0) = a$ and $x'(0) = b$,

```
>  sol := dsolve({eq,x(0)=a,D(x)(0)=b},x(t));
```

You can convert *sol* to a Maple function using the command **unapply** so you can plot this function,

```
>  x:= unapply(rhs(f),t);
```

Of course, you will need to define the values of $m$, $c$, $k$, $a$ and $b$ as well as the function $F(t)$ before you can plot.

(c) For problem 2, you can input the function $C(\omega)$ as a Maple function with two inputs $c$, and $\omega$,

```
> C:= (c,w) -> ____________;
```

You need to fill in the definition for $C(\omega)$ given in the text. Then, plot the three curves on the same graph by typing,

```
> plot([C(1/2,w), C(1,w), C(2,w)],w=0..10,color=[blue,red,black]);
```

To find where $C(\omega)$ reaches its maximum, solve for when is $C'(\omega) = 0$. For example for $c = 1/2$, you can type in these,

```
> DC:= diff(C(c,w),w);
> c:= 1/2;
> w:= solve(DC=0,w);
```

There will be multiple solutions to this problem. Pick the one that is relevant to the problem we are considering here. Then, evaluate C at this particular value of $\omega$.

```
> evalf(C(1/2,w1);
```