Math 1320-6 Lab 9

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Name: Answer Key

uNID: ________________________

<u>Instructions and due date:</u>

- **Due:** 21 April 2016 at the start of class.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).

1. Wind chill is the apparent decrease in air temperature felt by the body due to the flow of air over exposed skin. It can be approximated by a formula which depends on air temperature and wind speed, as follows:

$$\text{Wind Chill } (^\circ\text{F}) = f(T, V) = 35.74 + 0.6215T - 35.75V^{0.16} + 0.4275T(V^{0.16}).$$

The formula $f(T, V)$ is valid for temperatures at or below 50 °F and wind speeds above 3 mph.

(a) Let V be a fixed number, say $V_0 > 3$ mph. In your own words, what does the graph of the single-variable function $f(T, V_0)$ look like?

$$f(T, V_0) = 35.74 + 0.6215T - 35.75V_0^{0.16} + 0.4275T(V_0^{0.16})$$
$$= \underbrace{(35.74 - 35.75)V_0^{0.6}}_{\text{constant}} + \underbrace{(0.6215 + 0.4275V_0^{0.16})}_{\text{constant}}T$$

$\longrightarrow f(T, V_0)$ is linear

(b) Compute $\dfrac{\partial f(T, V_0)}{\partial T}$. What does it represent in the context of the graph of $f(T, V_0)$?

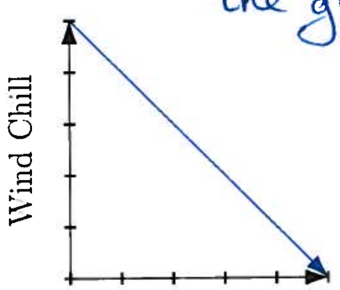
$$\frac{\partial f(T, V_0)}{\partial T} = 0.6215 + 0.4275V_0^{0.16}$$

Slope of the line

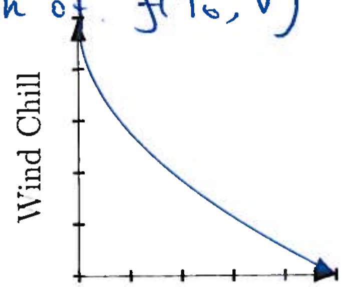
(c) Let T be a fixed number, say $T_0 \leq 50$ °F. Consider the single-variable function $f(T_0, V)$. Which of the three graphs below agrees (roughly) with the graph of $f(T_0, V)$?

$$f(T_0, V) = 35.74 + 0.6215T_0 - 35V^{0.16} + 0.4275T_0V^{0.1}$$
$$= (35.74 + 0.6215T_0) + \underbrace{(-35 + 0.4275T_0)}_{\text{This is negative unless } T_0 > 70 \text{ ... but } T_0 \leq 50 \text{ so it is always negative}}V^{0.1}$$

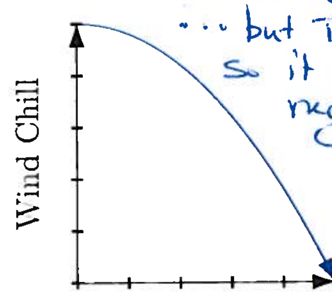
$\Rightarrow$ (b) roughly agrees with the graph of $f(T_0, V)$



(a)



(b)



(c)

2. Suppose that the temperature of a hot plate at a certain point in time can be modeled by $T(x, y) = 150e^{-3x^2-y^2}$ (in Fahrenheit).

   (a) Find an equation involving x and y for a level curve of $T(x, y)$. (Hints: Suppose that $0\,^\circ\mathrm{F} < C < 150\,^\circ\mathrm{F}$. Also, recall that a level curve is given implicitly by $T(x, y) = C$.)

$$150e^{-3x^2-y^2} = C, \quad 0 < C < 150$$

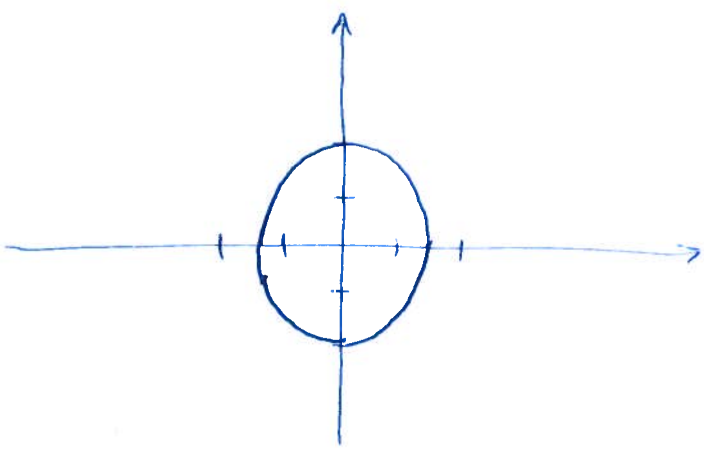
$$e^{-3x^2-y^2} = \frac{C}{150} \quad \Rightarrow \text{Ellipse}$$

$$\ln\left(e^{-3x^2-y^2}\right) = \ln\left(\frac{C}{150}\right)$$

$$-3x^2 - y^2 = \underbrace{\ln\left(\frac{C}{150}\right)}_{\text{This is negative}}$$

$$\longrightarrow \quad 3x^2 + y^2 = -\ln\left(\frac{C}{150}\right)$$

   (b) Let $C = 1$, and sketch the level curve.



$x = 0 \longrightarrow y = 1$
$y = 0 \longrightarrow x = \frac{1}{\sqrt{3}}$

   (c) What is the physical meaning of the level curves of $T(x, y)$?

Curves along which the temperature is constant.

3. Use polar coordinates to find the value of

$$\lim_{(x,y)\to(0,0)} \frac{\sin(3x^2+3y^2)}{x^2+y^2}.$$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2+y^2}$

$\lim_{(x,y)\to(0,0)} \iff \lim_{r\to 0^+}$

$$\lim_{r\to 0^+} \frac{\sin(3r^2)}{r^2}$$

$$= \lim_{r\to 0^+} 3 \cdot \frac{\sin(3r^2)}{3r^2}$$

$$= 3 \cdot \lim_{r\to 0^+} \frac{\sin(3r^2)}{3r^2}$$

$$= 3 \cdot 1$$

$$= 3$$

4. Laplace's equation is given by $u_{xx} + u_{yy} = 0$. It can be used to model the distribution of the temperature in an object (among other things). The two main assumptions required are: 1) there are no external sources of heat, and 2) the temperature is at steady-state ($\frac{\partial u}{\partial t} = 0$). Solutions of Laplace's equation are called *harmonic functions*.

If there are external sources of heat, represented by a function $f(x, y)$ (and if the temperature is at steady-state), then a different model is more appropriate: Poisson's equation, which is given by $u_{xx} + u_{yy} = f$.

(a) Is $u(x, y) = x^2 + y^2$ a harmonic function? If not, what does $f(x, y)$ need to be so that $u(x, y)$ is a solution of Poisson's equation?

$$u_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(x^2+y^2)\right) = \frac{\partial}{\partial x}(2x) = 2$$

$$u_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(x^2+y^2)\right) = \frac{\partial}{\partial y}(2y) = 2$$

$\rightarrow u_{xx} + u_{yy} = 4$

$\rightarrow u(x,y)$ is not a harmonic function.

Moreover, need $f(x,y) = 4$ for $u(x,y)$ to be a soln of Poisson's eqn

(b) Is $u(x, y) = (\sinh y)(\sin x)$ a harmonic function? If not, what does $f(x, y)$ need to be so that $u(x, y)$ is a solution of Poisson's equation?

$$u_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(\sinh y \sin x)\right) = \frac{\partial}{\partial x}(\sinh y \cos x) = -\sinh y \sin x$$

$$u_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(\sinh y \sin x)\right) = \frac{\partial}{\partial y}(\cosh y \sin x) = \sinh y \sin x$$

$\rightarrow u_{xx} + u_{yy} = 0$

$\rightarrow u(x,y)$ is a harmonic function.