Math 1320-6 Lab 9

Name: _____

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Instructions and due date:

- **Due:** 21 April 2016 at the start of class.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).

1. Wind chill is the apparent decrease in air temperature felt by the body due to the flow of air over exposed skin. It can be approximated by a formula which depends on air temperature and wind speed, as follows:

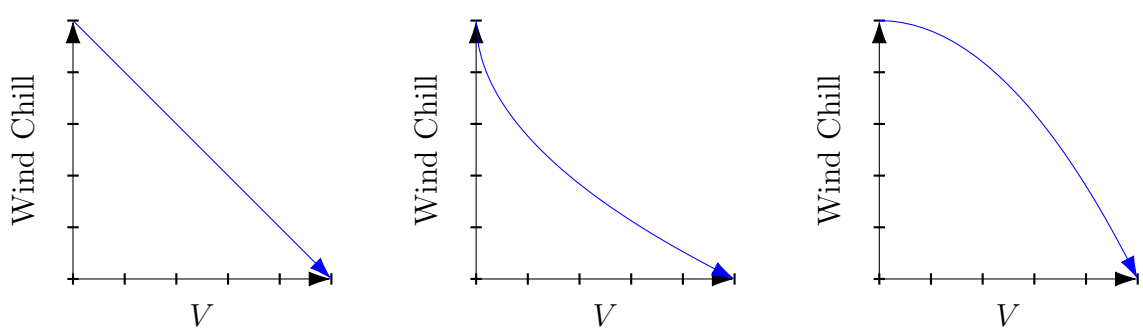
$$\text{Wind Chill } (^\circ\text{F}) = f(T, V) = 35.74 + 0.6215T - 35.75V^{0.16} + 0.4275T(V^{0.16}).$$

The formula $f(T, V)$ is valid for temperatures at or below $50\,^\circ$F and wind speeds above 3 mph.

(a) Let V be a fixed number, say $V_0 > 3$ mph. In your own words, what does the graph of the single-variable function $f(T, V_0)$ look like?

(b) Compute $\dfrac{\partial f(T, V_0)}{\partial T}$. What does it represent in the context of the graph of $f(T, V_0)$?

(c) Let T be a fixed number, say $T_0 \leq 50\,^\circ$F. Consider the single-variable function $f(T_0, V)$. Which of the three graphs below agrees (roughly) with the graph of $f(T_0, V)$?



(a) (b) (c)

2. Suppose that the temperature of a hot plate at a certain point in time can be modeled by $T(x, y) = 150e^{-3x^2-y^2}$ (in Fahrenheit).

   (a) Find an equation involving x and y for a level curve of $T(x, y)$. (Hints: Suppose that $0\,^\circ\text{F} < C < 150\,^\circ\text{F}$. Also, recall that a level curve is given implicitly by $T(x, y) = C$.)

   (b) Sketch a typical level curve $T(x, y) = C$.

   (c) What is the physical meaning of the level curves of $T(x, y)$?

3. Use polar coordinates to find the value of

$$\lim_{(x,y)\to(0,0)} \frac{\sin(3x^2+3y^2)}{x^2+y^2}.$$

4. Laplace's equation is given by $u_{xx} + u_{yy} = 0$. It can be used to model the distribution of the temperature in an object (among other things). The two main assumptions required are: 1) there are no external sources of heat, and 2) the temperature is at steady-state ($\frac{\partial u}{\partial t} = 0$). Solutions of Laplace's equation are called *harmonic functions*.

   If there are external sources of heat, represented by a function $f(x, y)$ (and if the temperature is at steady-state), then a different model is more appropriate: Poisson's equation, which is given by $u_{xx} + u_{yy} = f$.

   (a) Is $u(x, y) = x^2 + y^2$ a harmonic function? If not, what does $f(x, y)$ need to be so that $u(x, y)$ is a solution of Poisson's equation?

   (b) Is $u(x, y) = (\sinh y)(\sin x)$ a harmonic function? If not, what does $f(x, y)$ need to be so that $u(x, y)$ is a solution of Poisson's equation?