Math 1320 Lab8

Instructor: Bridget Fan

Name: ____________

Due: 04/14/16 in class

Show all work including necessary graphs to get full credits!

1. Given two curves $\mathbf{r}_1(t) = \langle t, 2-t, -3+t^2\rangle$ and $\mathbf{r}_2(s) = \langle 3-s, s-1, s^2\rangle$,

   (x,y,z)

   (a) At what point will the two curves intersect?

$$\begin{cases} t=3-s \\ 2-t=s-1 \Rightarrow t=1-s+2=3-s \\ -3+t^2=s^2 \Rightarrow -3+(3-s)^2=s^2 \Rightarrow -3=s^2-(3-s)^2=(s+3-s)(s-3+s)=3(2s-3) \end{cases}$$

$$\Rightarrow 2s-3=-1$$

$$2s=2$$

$$\left.\begin{matrix} s=1 \\ t=3-1=2 \end{matrix}\right\} \Rightarrow \text{Intersect at } (2,0,1)$$

   (b) Find their angle of intersection. (Hint: The angle between two curves is the angle between their tangent vectors at the intersection point.)

$$r_1'(t)=\langle 1,-1,2t\rangle \Rightarrow r_1'(2)=\langle 1,-1,4\rangle$$

$$r_2'(s)=\langle -1,1,2s\rangle \quad r_2'(1)=\langle -1,1,2\rangle$$

$$r_1'(2)\cdot r_2'(1) = |r_1'(2)|\cdot|r_2'(1)|\cos\theta$$

$$-1-1+8 = \sqrt{1+1+16}\cdot\sqrt{1+1+4}\cos\theta$$

$$\cos\theta = \frac{6}{3\sqrt{2}\cdot\sqrt{6}}$$

$$\Rightarrow \theta = \arccos\left(\frac{1}{\sqrt{3}}\right)$$

(c) Answer: No, they won't. Because the intersection happens at different time for the two cars.

2. The parameterization for a curve is given by

$$x = 9\cos t \qquad y = 4\sin t.$$

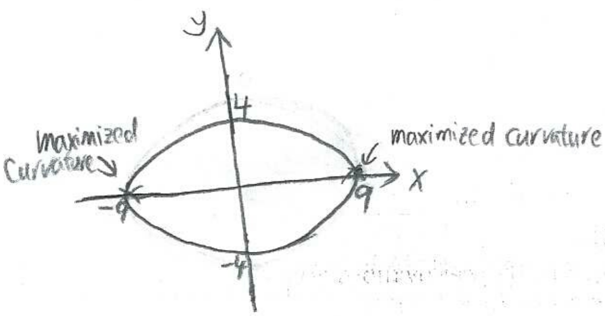
(a) By using the identity $\sin^2(t) + \cos^2(t) = 1$, express the function of curve in only x-y variable.

$\sin t = \frac{y}{4}$ $\Rightarrow \frac{x^2}{9^2} + \frac{y^2}{4^2} = 1$

$\cos t = \frac{x}{9}$

points

(b) Scratch the graph of the curve. Use a colored pen to label your guess of the point where the curvature is maximized and briefly explain why.



Because curvature measures how fast a curve is changing direction at a given point

(c) Find the expression for the curvature, $\kappa(t)$, of the curve.

$r'(t) = (-9\sin t, 4\cos t)$

$r''(t) = (-9\cos t, -4\sin t)$

$|r'(t)| = \sqrt{81\sin^2 t + 16\cos^2 t}$

$|r'(t) \times r''(t)| = \begin{vmatrix} -9\sin t & 4\cos t \\ -9\cos t & -4\sin t \end{vmatrix} = 36\sin^2 t + 36\cos^2 t = 36 \Rightarrow \kappa(t) = \frac{36}{(\sqrt{81\sin^2 t + 16\cos^2 t})^3}$

(d) At what point is $\kappa(t)$ maximized? Does the result agree with your guess in part (b).

$\kappa(t) = 36\,(81\sin^2 t + 16\cos^2 t)^{-\frac{3}{2}}$

$= 36\,(65\sin^2 t + 16)^{-\frac{3}{2}}$

$= \frac{36}{(65\sin^2 t + 16)^{\frac{3}{2}}}$

$\kappa(t)$ is maximized $\Leftrightarrow$ $(65\sin^2 t + 16)^{\frac{3}{2}}$ is minimized

$\Leftrightarrow$ $65\sin^2 t + 16$ is minimized $\Leftrightarrow$ $\sin^2 t$ is minimized

$\Leftrightarrow$ $\sin t = 0$ $\Leftrightarrow$ $t = 0, \pi$.

$t = 0$ $(-9, 0)$ ✓ same as guess

$t = \pi$ $(9, 0)$

3. The DNA molecule has the shape of a double helix (see Figure 3 on page 696 of textbook). The radius of each helix is about 10 angstroms ($1\text{Å} = 10^{-8}$cm). Each helix rises about 34Åduring each complete turn, and there are about 2.9×10^8 complete turns. Therefore the vector equation is

$$\mathbf{r}(t) = \left\langle 10\cos t, 10\sin t, \frac{34t}{2\pi} \right\rangle \quad \text{(measureing in angstroms)},$$

where t goes from 0 to $2.9 \times 10^8 \times 2\pi$. ~~Estimate~~ Compute the length of each helix. in centemiters

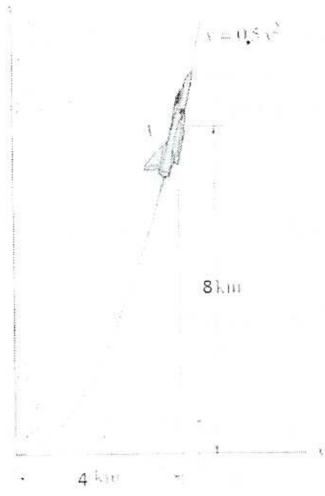
$$L = \int_0^{2.9\times10^8\times2\pi} \sqrt{100\sin^2 t + 100\cos^2 t + \left(\frac{34}{2\pi}\right)^2}\, dt$$

$$= 2.9\times10^8\times2\pi \times \sqrt{100+\left(\frac{34}{2\pi}\right)^3}$$

$$= 2.07\times10^{10}\ \text{Å}$$

$$= 207\ \text{cm}$$

4. A jet plane travels along a vertical parabolic path defined by the equation y $0.5x^2$.At point A, the jet has a speed of $250 m/s$, which is increasing at the rate of $0.8 m/s^2$.



(a) What is tangential component of the total acceleration?

$a_t = 0,8 m/s^2$

(b) What is the radius of curvature of the path at A?

$f'(x) = x \qquad f'(4) = 4$

$f''(x) = 1 \qquad f''(4) = 1$

$$k = \frac{|f''(x)|}{(1+[f'(x)]^2)^{\frac{3}{2}}}$$

$$= \frac{1}{(1+16)^{\frac{3}{2}}} = 17^{-\frac{3}{2}} \text{ km}^{-1} = 17^{-\frac{3}{2}} \times 10^{-3} \text{ m}^{-1}$$

(c) What is the normal component of acceleration?

$$a_n = v^2 \cdot k = 250^2 (17)^{-\frac{3}{2}} \times 10^{-3} = 0,8917 \ m/s^2$$

(d) Use your answer from previous parts. calculate the magnitude of the acceleration vector.

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0,8^2 + 0,8917^2} = 1,198 \ m/s^2$$