Math 1320-6 Lab 8

Name: ____________________

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Instructions and due date:

- **Due:** 14 April 2016 at the start of class.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).

1. Consider two curves: $\mathbf{r}_1(t) = \langle t, 2-t, -3+t^2 \rangle$ and $\mathbf{r}_2(s) = \langle 3-s, s-1, s^2 \rangle$.

   (a) At what point (x, y, z) will the two curves intersect?

   (b) Find their angle of intersection. (Hint: The angle between two curves at an intersection point is the angle between their tangent vectors at that point.)

   (c) Suppose that $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ describe the paths of two cars, where t represent time. Based your calculation in part (a), will the two cars crash into each other? Why or why not?

2. The parametric equations for a curve are given by

$$x = 9\cos t \qquad y = 4\sin t.$$

(a) Use the identity $\sin^2(t) + \cos^2(t) = 1$ to write down a single equation for the curve, in only the x and y variables.

(b) Sketch the graph of the curve. Use a colored pen to label your guess of the points where the curvature is maximized and briefly explain why.

(c) Find the expression for the curvature, $\kappa(t)$, of the curve.

(d) At what points are $\kappa(t)$ maximized? Do your results agree with your guess in part (b)?

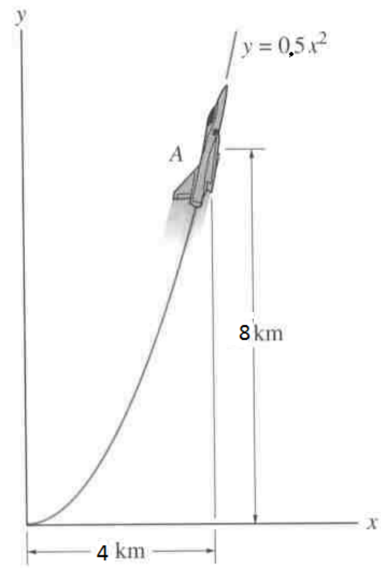
3. The DNA molecule has the shape of a double helix (see Figure 3 on page 696 of textbook). The radius of each helix is about 10 angstroms ($1\,\text{Å} = 1 \times 10^{-8}\,\text{cm}$). Each helix rises about $34\,\text{Å}$ during each complete turn, and there are about 2.9×10^8 complete turns. Therefore the vector equation is

$$\mathbf{r}(t) = \left\langle 10\cos t, 10\sin t, \frac{34t}{2\pi} \right\rangle \quad \text{(measured in angstroms)},$$

where t goes from 0 to $2.9 \times 10^8 \times 2\pi$. Compute the length of each helix, in centimeters.

4. A jet plane travels along a vertical parabolic path defined by the equation $y = 0.5x^2$ (x and y both have units of kilometers). At point A, the jet has a speed of $250\,\mathrm{m\,s^{-1}}$; the speed is increasing at the rate of $0.8\,\mathrm{m\,s^{-2}}$.

   (a) What is tangential component of the total acceleration, at the point A?



   (b) What is the curvature of the path, at the point A?

   (c) What is the normal component of acceleration, at the point A? (Be careful with units.)

   (d) Use your answer from the previous parts to calculate the magnitude of the acceleration vector, at the point A. (Be careful with units.)