Math 1320-6 Lab 7

T.A.: Kyle Steffen

10 March 2016

Name: Answer Key

uNID: ______

Instructions and due date:

- **Due:** 24 March 2016 at the start of class.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).

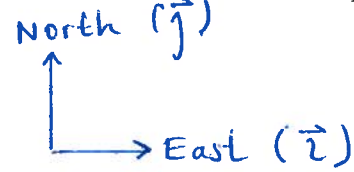
1. This question is an application of vector addition:

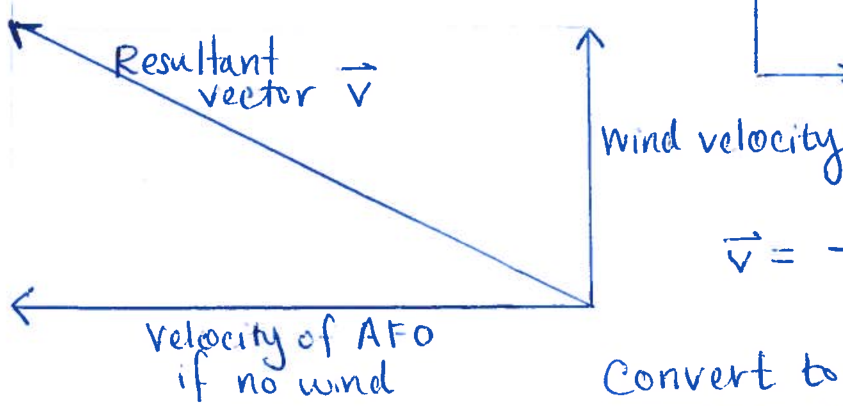
The call sign for the airplane that carries the President of the United States is “Air Force One” (AFO). It has a cruising speed of around $925\,\mathrm{km\,h^{-1}}$.

At one point in time during a flight from Washington, DC to Los Angeles, California, suppose that: a) the heading of AFO was due west, b) its speed was 90% of its cruising speed, and c) the wind was due north with a speed of $25\,\mathrm{m\,s^{-1}}$.

(*Note:* The “heading” of an aircraft refers to the direction in which its nose is pointing, and thus the direction of its motion if there were no winds.)

(a) Draw a sketch of the three vectors: i.) the velocity of AFO if there were no wind; ii.) the wind velocity; and iii.) the resultant vector, which gives the velocity of AFO as seen by a stationary observer on the ground. Also: What is the resultant vector in component form?





$\vec{v} = -(0.9)(925\,\frac{km}{h})\vec{i} + (25\,\frac{m}{s})\vec{j}$

Convert to $\frac{m}{s}$:

$\vec{v} = (-231.25\,\frac{m}{s})\vec{i} + (25\,\frac{m}{s})\vec{j}$

$\longrightarrow \boxed{\vec{v} = \langle -231.25, 25 \rangle \frac{m}{s}}$

(b) What direction and speed was AFO moving at that point in time?

Speed $= |\vec{v}| = \sqrt{(-231.25)^2 + (25)^2}$

$\longrightarrow \boxed{\text{Speed} \approx 232.60\,\frac{m}{s}}$

Direction: $\tan\theta = \dfrac{25}{-231.25}$

$\theta = \arctan\left(\dfrac{25}{-231.25}\right) + \pi k$

with $k = 0, \pm 1, \pm 2$

(since tan is π-periodic)

Recall:

Range(arctan) $= (-\frac{\pi}{2}, \frac{\pi}{2})$

$\longrightarrow$ 1st or 4th quadrant

Our vector is in the 2nd quadrant

$\rightarrow \boxed{\text{Direction} = \theta = \arctan\left(\dfrac{25}{-231.25}\right) + \pi \approx 173.83^\circ \text{ from positive x-axis}}$

$\boxed{\text{Alt: } 180^\circ - 173.83 \approx 6.17^\circ \text{ North of West}}$

2. Consider the following four vectors: $\mathbf{a} = \langle 1, 1\rangle$, $\mathbf{b} = \langle 1, -1\rangle$, $\mathbf{v} = \langle -3, 4\rangle$, and $\mathbf{w} = \langle x, y\rangle$.

(a) Compute $\text{proj}_{\mathbf{a}}\,\mathbf{v}$ (the projection of $\mathbf{v}$ onto $\mathbf{a}$) and $\text{proj}_{\mathbf{b}}\,\mathbf{v}$. State your answers in component form.

$$\text{proj}_{\vec{a}}\vec{v} = \frac{\vec{v}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} = \frac{(-3)(1)+(4)(1)}{(\sqrt{1^2+1^2})^2}\langle 1,1\rangle$$
$$= \frac{1}{2}\langle 1,1\rangle = \boxed{\left\langle \frac{1}{2}, \frac{1}{2}\right\rangle}$$

$$\text{proj}_{\vec{b}}\vec{v} = \frac{\vec{v}\cdot\vec{b}}{|\vec{b}|^2}\vec{b} = \frac{(-3)(1)+(4)(-1)}{(\sqrt{1^2+1^2})^2}\langle 1,-1\rangle$$
$$= (-7/2)\langle 1,-1\rangle = \boxed{\left\langle \frac{-7}{2}, \frac{7}{2}\right\rangle}$$

(b) Compute $\text{proj}_{\mathbf{a}}\,\mathbf{w}$ and $\text{proj}_{\mathbf{b}}\,\mathbf{w}$. State your answers in component form.

$$\text{proj}_{\vec{a}}\vec{w} = \frac{\vec{w}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} = \frac{1}{2}(x\cdot 1 + y\cdot 1)\langle 1,1\rangle$$
$$= \boxed{\left\langle \frac{x+y}{2}, \frac{x+y}{2}\right\rangle}$$

$$\text{proj}_{\vec{b}}\vec{w} = \frac{\vec{w}\cdot\vec{b}}{|\vec{b}|^2}\vec{b} = \frac{1}{2}(x\cdot 1 + y\cdot(-1))\langle 1,-1\rangle$$
$$= \langle (x-y)/2, -(x-y)/2\rangle = \boxed{\left\langle \frac{x-y}{2}, \frac{y-x}{2}\right\rangle}$$

(c) Use your answers from part (b), and compute the following sum: $\text{proj}_{\mathbf{a}}\,\mathbf{w} + \text{proj}_{\mathbf{b}}\,\mathbf{w}$. Simplify your answer.

$$= \left\langle \frac{x+y}{2} + \frac{x-y}{2}, \frac{x+y}{2} + \frac{y-x}{2}\right\rangle$$
$$= \left\langle \frac{x}{2} + \cancel{\frac{y}{2}} + \frac{x}{2} - \cancel{\frac{y}{2}}, \cancel{\frac{x}{2}} + \frac{y}{2} + \frac{y}{2} - \cancel{\frac{x}{2}}\right\rangle = \boxed{\langle x, y\rangle}$$

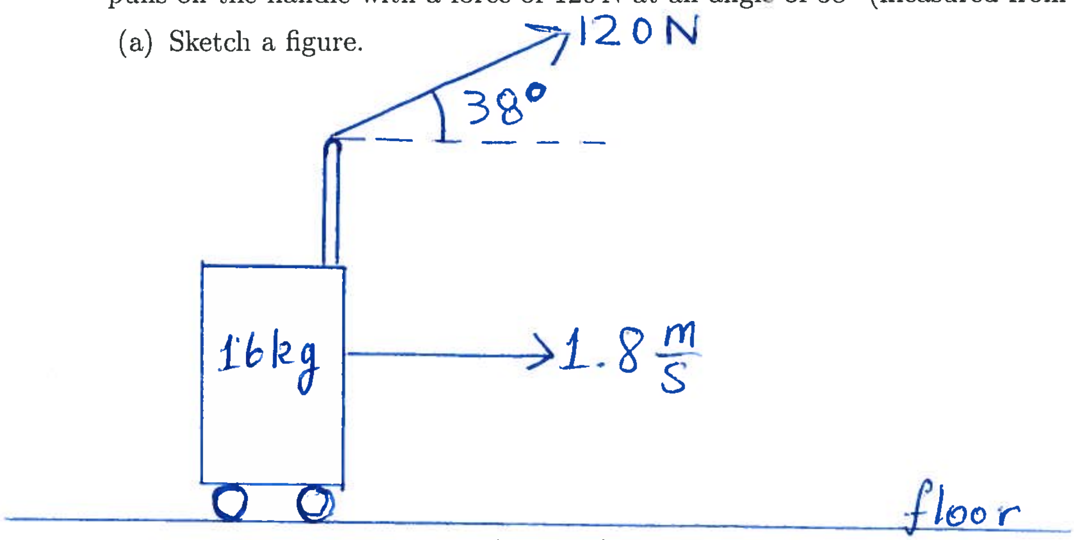
(d) Your answer to part (c) suggests the following: *"any vector $\langle x, y\rangle$ in $\mathbb{R}^2$ can be expressed as the sum of its projections onto two distinct vectors."* This is not always true. If we require that the two distinct vectors have a non-zero angle between them, then it is always true. For part (d): What is the angle between $\mathbf{a}$ and $\mathbf{b}$?

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{1(1)+1(-1)}{2\cdot 2} = \frac{1-1}{4} = 0$$

$$\longrightarrow \cos\theta = 0 \longrightarrow \boxed{\theta = \frac{\pi}{2}, \text{ or } 90^\circ}$$

3. Terry has under an hour to check her bag, get through airport security, and catch her flight. She is speed walking to her airline's front desk, and is pulling her 16 kg suitcase at a constant speed of $1.8\,\mathrm{m\,s^{-1}}$. (Assume that the suitcase is standing upright and does not lean at an angle.) She pulls on the handle with a force of 120 N at an angle of 38° (measured from the horizontal).

(a) Sketch a figure.



(b) Calculate the work done (in N · m) when Terry pulls her suitcase a distance of 100 m.

$$\text{Work} = \text{Force} \cdot \text{Displacement}$$

$$= |\vec{F}||\vec{D}|\cos\theta$$

$$= (120\text{N})(100\text{ m})\cos(38°)$$

$$\rightarrow W \approx 9456.13 \text{ N}\cdot\text{m}$$

— or —

$$\boxed{W \approx 9456.13 \text{ Joules}}$$

4. A crane boom is 30 m long, has a mass of 450 kg, and is supporting a load of 10 kN, as shown in the two diagrams below.

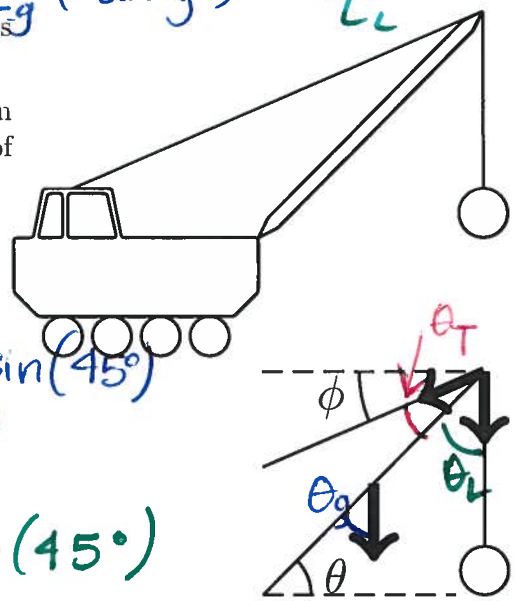
   Three forces acting on the boom generate torque at its pivot point: i.) the force due to gravity (acting at the center of mass of the boom), ii.) the force due to the tension in the cable (acting at the upper end of the boom), and iii.) the force due to the load (also acting at the upper end of the boom).

   The boom is stationary, which means: "torque due to gravity + torque due to the load = torque due to the tension."

   $\vec{\tau}_T$ $\qquad$ $\vec{\tau}_g$ ("tau-g") $\qquad$ $\vec{\tau}_L$

   (a) Write down a single equation which involves the magnitude of the three torques.

   (Hint: There should be exactly one unknown variable in your equation—the magnitude of the force due to the tension.)



$\theta = 45°$ and $\phi = 23°$

$$|\vec{\tau}_g| = |\vec{F}_g||\vec{r}_g| \sin(\theta_g)$$
$$= \left(450\text{kg} \cdot 9.8 \tfrac{\text{m}}{\text{s}^2}\right)\left(\tfrac{30\text{m}}{2}\right)\sin(45°)$$
$$= 450 \cdot 9.8 \cdot 15 \cdot \tfrac{\sqrt{2}}{2} \text{ N}\cdot\text{m}$$
$$\vec{\tau}_L = |\vec{F}_L||\vec{r}_L| \sin(\theta_L)$$
$$= (10000\text{ N})(30\text{ m})\sin(45°)$$
$$= 10000 \cdot 30 \cdot \tfrac{\sqrt{2}}{2} \text{ N}\cdot\text{m}$$

$\theta_g = \theta_L = 45°$

$\theta_T = \theta - \phi = 22°$

   (b) Solve the equation from part (a) to find the magnitude of the tension in the cable (in Newtons).

$$\vec{\tau}_T = |\vec{F}_T||\vec{r}_T| \sin(\theta_T)$$
$$= |\vec{F}_T|(30\text{ m})\sin(22°)$$
$$= \underbrace{|\vec{F}_T|}_{\text{unknown}} \cdot 30 \cdot \sin(22°) \text{ N}\cdot\text{m}$$

→ Equation for part (a) is:

$$|\vec{\tau}_g| + |\vec{\tau}_L| = |\vec{\tau}_T|$$

$$\boxed{\left(450 \cdot 9.8 \cdot 15 \cdot \tfrac{\sqrt{2}}{2} + 10000 \cdot 30 \cdot \tfrac{\sqrt{2}}{2}\right) = |\vec{F}_T| \cdot 30 \cdot \sin(22°)}$$

Answer for (a)

For (b), we solve for $|\vec{F}_T|$:

$$|\vec{F}_T| = \frac{450 \cdot 9.8 \cdot 15 \cdot \frac{\sqrt{2}}{2} + 10000 \cdot 30 \cdot \frac{\sqrt{2}}{2}}{30 \cdot \sin(22°)}$$

$\longrightarrow |\vec{F}_T| \approx 23038.14$

$\boxed{\Rightarrow \text{The tension in the cable is} \approx 23038.14 \text{ Newtons}}$

Answer for (b)