Math 1320-6 Lab 6

Name: ______________________

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uNID: ______________________

<u>Instructions and due date:</u>

- **Due:** 10 March 2016 at the start of class.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).

1. Use the Alternating Series Estimation Theorem or Taylor's Inequality to answer the following questions:

   (a) Approximate the function $f(x) = \ln(1 + 2x)$ by a Taylor polynomial of degree 3 at $x = 1$. How accurate is the estimation when $0.4 \leq x \leq 1.6$?

   (b) Approximate the function $f(x) = \arctan x$ by a Taylor polynomial of degree 5 at $x = 0$. Estimate the range of x such that the accuracy of the estimation is around 0.05.

2. **Maclaurin Series** Consider the differential equation $\frac{dy}{dt} = ky^2$.
   (a) Write the first few terms of the Maclaurin expansion for $y(t)$.

   (b) Suppose that $y^{(n)}(t) = n!k^n y^{n+1}(t)$. Show that $y^{(n+1)}(t) = (n+1)!k^{n+1}y^{n+2}(t)$.

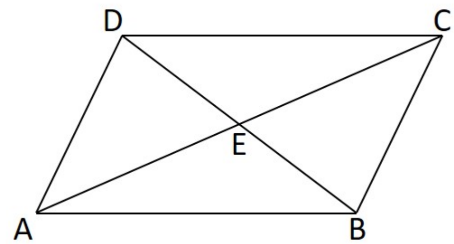
   (c) The argument you have just used is called *induction*. If for the base case $n = 1$ some relation holds, and assuming the relation holds for $n = m$ implies that the relation holds for $n = m + 1$, then it is true for all n. Write the Maclaurin series of $y(t)$.

   (d) Observe the pattern in the Maclaurin series. Given $y(0)$, what is the solution of the differential equation? Use what you know about geometric series to find the time $t > 0$ for which $y(t)$ ceases to exist.

3. Find the equation for the following objects in $\mathbb{R}^3$:
   (a) A sphere, if one of its diameters has endpoints $(0, 3, 2)$ and $(7, -1, -2)$.

   (b) The set of points that are equidistant from $A(-1, 0, 3)$ and $B(1, 1, -1)$.

4. (a) Given the parallelogram:



Find $\overrightarrow{AB} + \overrightarrow{AD} - \overrightarrow{DC}$.

(b) Compute $|\vec{a}|$. Also, find the unit vector that pointing in the same direction as $\vec{a} = 2i - j$.

(c) Repeat part (b) for the vector $\vec{a} = \langle 1, 2, 3 \rangle$.