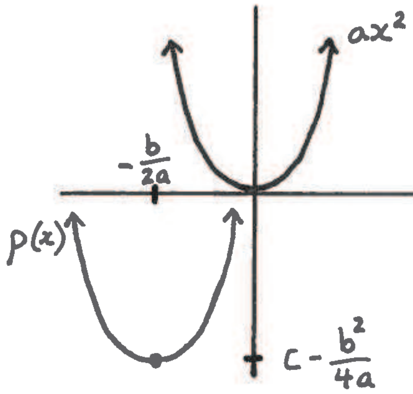
Number of roots of a quadratic polynomial

Let $p(x) = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}$ and $a \neq 0$.

We can "complete the square" to rewrite $p(x)$ as

$p(x) = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$.



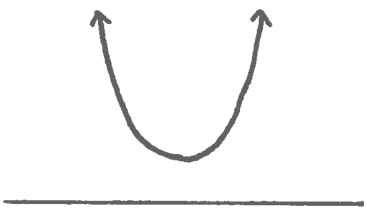
Either the parabola for $p(x)$ "opens up" ($\cup$, $a > 0$) or it "opens down" ($\cap$, $a < 0$).

Either the vertex of the parabola for $p(x)$ is above the x-axis ($c - \frac{b^2}{4a} > 0$), on the x-axis ($c - \frac{b^2}{4a} = 0$), or below the x-axis ($c - \frac{b^2}{4a} < 0$).

There are $2 \cdot 3 = 6$ total options for the graph of $p(x)$.

---

①



$a \underline{\qquad} 0$

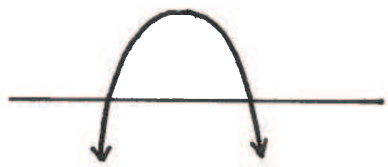
$c - \frac{b^2}{4a} \underline{\qquad} 0$

$4a \underline{\qquad} 0$

$4ac - b^2 \underline{\qquad} 0$

$b^2 - 4ac \underline{\qquad} 0$

②



$a \underline{\qquad} 0$

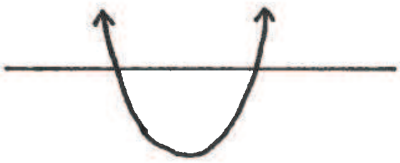
$c - \frac{b^2}{4a} \underline{\qquad} 0$

$4a \underline{\qquad} 0$

$4ac - b^2 \underline{\qquad} 0$

$b^2 - 4ac \underline{\qquad} 0$

③



a ___ 0

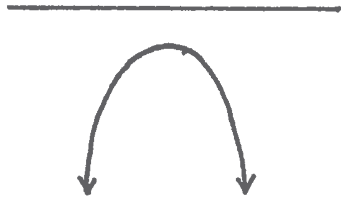
$c - \frac{b^2}{4a}$ ___ 0

$4a$ ___ 0

$4ac - b^2$ ___ 0

$b^2 - 4ac$ ___ 0

④



a ___ 0

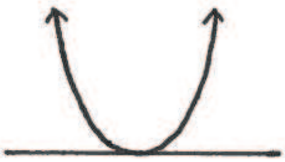
$c - \frac{b^2}{4a}$ ___ 0

$4a$ ___ 0

$4ac - b^2$ ___ 0

$b^2 - 4ac$ ___ 0

⑤

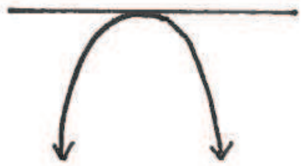


$c - \frac{b^2}{4a}$ ___ 0

$4ac - b^2$ ___ 0

$b^2 - 4ac$ ___ 0

⑥



$c - \frac{b^2}{4a}$ ___ 0

$4ac - b^2$ ___ 0

$b^2 - 4ac$ ___ 0

---

number of roots of $p(x) = ax^2 + bx + c$	$b^2 - 4ac$
0	
1	
2	

$b^2 - 4ac$ is the <u>discriminant</u> of $ax^2 + bx + c$.