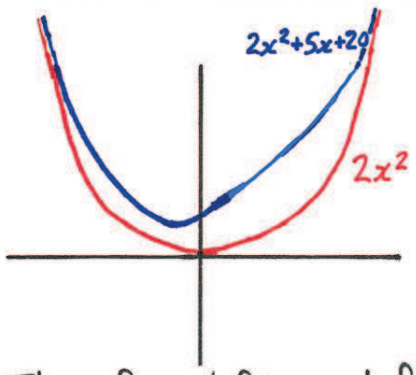
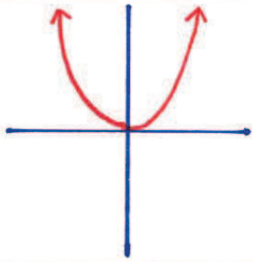
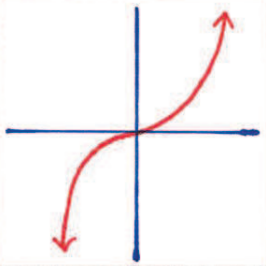
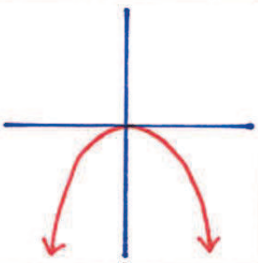
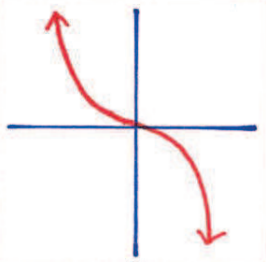
Asymptotic behavior of polynomials

x	$2x^2$	$5x$	20	$2x^2+5x+20$
0	0	0	20	20
1	2	5	20	27
10	200	50	20	270
100	20,000	500	20	20,520
1 million	2 trillion	5 million	20	about 2 trillion

If x is a very large number (positive or negative) then $2x^2 \approx 2x^2+5x+20$. More generally, if x is a very large number, then $a_n x^n \approx a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.



The far left and far right part of the graph of $2x^2+5x+20$ is very close to the graph of $2x^2$.

Graph of ax^n	n even	n odd
$a>0$		
$a<0$		

Graphing Polynomials

Step 1: Plot the roots (if there are any) of your polynomial $p(x)$.

Step 2: For each consecutive pair of roots, choose a number between them. Call that chosen number b. Check to see if $p(b)$ is positive or negative. Draw a dot for the point in the graph given by $(b, p(b))$.

Step 3: The far left and right part of the graph of $p(x)$ is very close to the graph of its leading term. So to the left and right of everything you've drawn so far, just draw the graph of the leading term. That's close enough.

Step 4: Connect the pieces from 1,2,3, but don't violate the vertical line test, and don't create any new x-intercepts.

$p(x) = 3(x+4)(x-2)(x^2+1)$

① Roots: ____ and ____

② Pick a number between the two roots: ____
Plug this number into $p(x)$.
Answer is positive / negative.

③ Leading term of $p(x)$: ____
Graph of leading term: