

## Graph Transformations

On this page,  $d > 0$  and  $c > 1$ .  $f(x)$  is a function whose graph we know.

New function	How points in graph of $f(x)$ become points in graph of new function.	How the overall picture changes
$f(x) + d$	$(a, b) \mapsto (a, b + d)$	shift up by $d$
$f(x) - d$	$(a, b) \mapsto (a, b - d)$	shift down by $d$
$cf(x)$	$(a, b) \mapsto (a, cb)$	stretch vertically by $c$
$\frac{1}{c}f(x)$	$(a, b) \mapsto (a, b/c)$	shrink vertically by $c$
$-f(x)$	$(a, b) \mapsto (a, -b)$	flip over $x$ -axis

↑  
change occurs  
"after  $f$ "

↑  
change occurs in second  
coordinate

↑  
change is vertical

↓  
change occurs  
"before  $f$ "

↓  
change occurs in first  
coordinate

↓  
change is horizontal

$f(x + d)$	$(a, b) \mapsto (a - d, b)$	shift left by $d$
$f(x - d)$	$(a, b) \mapsto (a + d, b)$	shift right by $d$
$f(cx)$	$(a, b) \mapsto (a/c, b)$	shrink horizontally by $c$
$f(x/c)$	$(a, b) \mapsto (ca, b)$	stretch horizontally by $c$
$f(-x)$	$(a, b) \mapsto (-a, b)$	flip over $y$ -axis

↑ change in these columns may seem counterintuitive for this second grouping of five functions. ↓

On this page,  
 $f: [-2, 2) \rightarrow \mathbb{R}$   
 $f(x) = x^2$

