Math 1050–4 Midterm Exam #3
April 17, 2015

• No notes, books, etc. are allowed.
• Answer the questions in the spaces provided.
• You have 50 minutes to complete this exam.
• Good luck!

Name: __________________________

For #1–7, are the statements true or false?

1. (1 point) \( \log_a(zw) = \log_a(z) + \log_a(w) \) 1. True

2. (1 point) \( a^x a^y = a^{x+y} \) 2. False

3. (1 point) \( \log_a(z^w) = w \log_a(z) \) 3. False

4. (1 point) \( (a^x)^y = a^{x+y} \) 4. False

5. (1 point) \( \log_a\left(\frac{z}{w}\right) = \frac{\log_a(z)}{\log_a(w)} \) 5. False

6. (1 point) \( \frac{a^x}{a^y} = a^{x-y} \) 6. True

7. (1 point) A linear equation in two variables \( (ax + by = r) \) has two solutions. Three possibilities: zero, one, or infinitely many solutions. 7. False

For #8–10, write the expression as a rational number in standard form.

8. (1 point) \( 3^{-500} \cdot 5^{20} \cdot 3^{-23} \)

\[
3^{-500} \cdot 5^{20} \cdot 3^{-23} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}
\]
8. \( \frac{1}{27} \)

9. (1 point) \( (5^3)^\frac{2}{3} \)

\[
5^{\frac{2}{3}}(\frac{3}{9}) = 5^3 = 125
\]
9. 125

10. (1 point) \( 100,000^{\frac{3}{5}} \)

\[
(10^5)^{\frac{3}{5}} = 10^{3} = 1,000
\]
10. 1,000

Points earned: ____________
out of a possible 10 points
For #11–12, write the expression as a rational number in standard form.

11. (1 point) \( \log_{10} \left( \frac{1}{1,000} \right) \)

\[
= \log_{10} (10^{-3}) = -3
\]

12. (1 point) \( \log_{4} \left( \frac{1}{\sqrt[4]{64}} \right) \)

\[
\log_{4} \left( \frac{1}{\sqrt[4]{64}} \right) = \log_{4} \left( \frac{1}{4^{3/4}} \right) = \log_{4} \left( 4^{-3/4} \right) = -\frac{3}{4}
\]

13. (1 point) Solve for \( x \) if \( \log_{8}(x) = -2 \). (Write your answer as a rational number in standard form.)

\[
\begin{align*}
x &= 8^{-2} \\
x &= \frac{1}{8^2} \\
x &= \frac{1}{64}
\end{align*}
\]
14. (1 point) Solve for $x$ if $2e^x - 5 = 3$.

\[
2e^x = 3 + 5 \\
2e^x = 8 \\
e^x = \frac{8}{2} \\
e^x = 4 \\
x = \log_e(4)
\]

15. (1 point) Solve for $x$ if $e^{11 - 2x} = \frac{3}{e^{-4x}}$

\[
e^{11 - 2x} (e^{-4x}) = \frac{3}{e^{-4x}} (e^{-4x}) \\
e^{11 - 2x - 4x} = 3 \\
e^{11 - 6x} = 3 \\
\rightarrow \quad 11 - 6x = \log_e(3)
\]

16. (1 point) Solve for $x$ if $|7x - 5| < 4$

\[-4 < 7x - 5 < 4\]

\[
\rightarrow -4 < 7x - 5 \\
-4 + 5 < 7x \\
1 < 7x \\
\frac{1}{7} < x
\]
17. (5 points) Graph $p(x) = \frac{-3}{5}(x - 1)(x - 1)(x + 2)(x + 2)(x^2 + 9)$

\[\chi = 0 \rightarrow p(0) = \frac{-3}{5}(-1)(-1)(2)(2)(4)\]

\[\rightarrow p(0) < 0\]

18. (5 points) Graph $r(x) = \frac{-8(x - 1)(x^2 + 7)}{5(x - 3)(x + 1)(x + 1)}$

Leading term:

\[-\frac{8}{5}x^3\]

\[x = 0 \rightarrow r(0) = \frac{-8(1)(7)}{5(-3)(1)(1)} < 0\]

\[x = 2 \rightarrow r(2) = \frac{-8(1)(4 + 7)}{5(-1)(3)(3)} > 0\]
For #22-25, graph the function, draw and label its x- and y-intercepts (if any), its vertical asymptotes (if any), and one additional point.

19. (3 points) \( \log_e(x - 1) \)

20. (4 points) \( e^{x-1} \)

21. (3 points) Graph \( g : \mathbb{R} \rightarrow \mathbb{R} \) where

\[
g(x) = \begin{cases} 
-x - 2 & \text{if } x \in (-\infty, 0) \\
x + 2 & \text{if } x \in [0, \infty) 
\end{cases}
\]
22. (a) (1 point) What is the slope of the line $9x - 5y = 1$?

\[
\begin{align*}
9x - 1 &= 5y \\
\frac{9x - 1}{5} &= y
\end{align*}
\]

\[a \frac{9}{5} - \frac{1}{5}
\]

(b) (2 points) Is there a unique solution to the following system?

\[
\begin{align*}
3y &= 17 + 6x \\
y &= \frac{17 + 6x}{3}
\end{align*}
\]

\[
\begin{align*}
y &= \frac{17}{3} + \frac{2x}{3}
\end{align*}
\]

23. (2 points) The following system of two linear equations has a unique solution. Find the solution. (Your answer should be a rational number in standard form.)

\[
\begin{align*}
2x + y &= 9 \\
-2x + 3y &= 11
\end{align*}
\]

\[
\begin{align*}
x &= \frac{9 - (5)}{2} \\
x &= \frac{4}{2} \\
x &= 2
\end{align*}
\]

23. $x = 2 \quad \frac{1}{5}, \quad y = 5$