True/False

1. (8 points) For #1–8, write the word "True" or "False."
   (a) \( \sqrt{x+y} = \sqrt{x} + \sqrt{y} \)  
   (b) \((xy)^n = x^n y^n\)  
   (c) \(a(b+c) = ab + ac\)  
   (d) \((x+y)^n = x^n + y^n\)  
   (e) \(\sqrt{xy} = \sqrt{x} \sqrt{y}\)  
   (f) \(\left( \frac{x}{y} \right)^n = \frac{x^n}{y^n}\)  
   (g) \(\sqrt[3]{x} \sqrt[3]{y} = \sqrt[3]{xy}\)  
   (h) \(-7x^5 + 3x^4 - 2x + 1\) has 7 roots

Algebra

2. (1 point) Find \(x\) where \((x + 16)^3 + 9 = -18\). (Your answer should be an integer in standard form.)
   \[ (x+16)^3 + 9 = -18 \]
   \[ x+16 = -3 \]
   \[ x = -19 \]

3. (1 point) If \(g(x)\) is an invertible function, and \(g(-3) = 5\), then what is \(g^{-1}(5)\)?
   \[ g(-3) = 5 \]
   \[ -3 = g^{-1}(5) \]

Points earned: _______
out of a possible 10 points
4. (1 point) Find the inverse of \( f(x) = 9\sqrt{x} - 1 + 2 \). (You can check your answer by seeing if \( f^{-1} \circ f(x) = x \).

\[
\begin{align*}
\frac{y - 2}{3} &= \sqrt{x - 1} \\
\left(\frac{y - 2}{3}\right)^2 &= x - 1 \\
\left(\frac{y - 2}{3}\right)^2 + 1 &= x
\end{align*}
\]

\[f^{-1}(y) = \left(\frac{y - 2}{3}\right)^2 + 1\]

5. (1 point) What is the implied domain of \( g(x) = -\sqrt[4]{4x - 9} + 3x^2 - 2 \)? (Write your answer as an interval.)

\[
\begin{align*}
4x - 9 < 0 : & \text{ values of } x \text{ for which } g(x) \text{ does not make sense} \\
4x < 9 : & x < \frac{9}{4} \\
\text{or } x \in (-\infty, \frac{9}{4})
\end{align*}
\]

\[
\begin{align*}
\mathbb{R} = (-\infty, \frac{9}{4}) : & \text{ all values of } x \text{ for which } g(x) \text{ does make sense (i.e., its implied domain)} \\
equivalently: & \left[\frac{9}{4}, \infty\right)
\end{align*}
\]

6. (1 point) Suppose that \( a \neq 0 \) and that \( b^2 - 4ac \geq 0 \). Write the following as an integer in standard form:

\[
\begin{align*}
\text{This is } p(x) \text{ evaluated at its root} \\
a \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2 + b \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) + c &= 0
\end{align*}
\]

\[
\text{This is a root of the polynomial } p(x) = ax^2 + bx + c
\]

6. \[
\boxed{0}
\]

Page 2 Points earned: ________ out of a possible 3 points
7. (1 point) Find \[ \frac{8x^4 + 6x^3 - 6x^2 - 2x + 4}{2x^2 + 1} \]

\[
\begin{array}{c|ccccc}
& 4x^2 & 3x & -5 \\
\hline
2x^2 + 1 & 8x^4 & +6x^3 & -6x^2 & -2x & +4 \\
- (8x^2 & +4x^2 & ) & \\
& 6x^3 & -10x & -2x & +4 \\
- (6x^3 & +3x & ) & \\
& -10x^2 & -5x & +4 & \\
- (-10x^2 & -5) & \\
& -5x & +9 & \\
\end{array}
\]

Solution to #7: \[ 4x^2 + 3x - 5 + \frac{-5x + 9}{2x^2 + 1} \]

8. (1 point) Find \[ \frac{3x^3 + 7x^2 - 2}{x + 3} \]

\[ = \frac{3x^3 + 7x^2 + 0x - 2}{x - (-3)} \]

\[ = -3 \begin{pmatrix} 3 & 7 & 0 & 1 & -2 \\ -9 & 6 & 1 & -18 \\ 3 & -2 & 6 & 1 & -20 \end{pmatrix} \]

9. (1 point) What is the slope of the straight line in \( \mathbb{R}^2 \) that passes through the point \((1, 3)\) and \((6, 13)\)?

\[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 3}{6 - 1} = \frac{10}{5} = 2 \]

9. \[ 2 \]

Page 3

Points earned: \[ \_\_\_\_ \]
out of a possible \( 3 \) points
10. (1 point) Complete the square: Write $6x^2 - 6x + 5$ in the form $\alpha(x + \beta)^2 + \gamma$, where $\alpha, \beta, \gamma \in \mathbb{R}$.
\[
\alpha = a, \quad \beta = b/2a, \quad \gamma = c - b^2/4a
\]
\[
\begin{align*}
\alpha &= 6, \\
\beta &= -6/2(6) = -1/2, \\
\gamma &= 5 - (-6)^2/4(6) = 5 - 36/4(6) = 5 - 6
\end{align*}
\]
\[
\therefore \quad 6(x - 1/2)^2 = 0
\]
10. 

11. (1 point) How many roots does $-5x^2 + 6x - 3$ have?
\[
b^2 - 4ac
\]
\[
= (6)^2 - 4(-5)(-3)
\]
\[
= 36 - 4(5)(3)
\]
\[
= 36 - 20(3)
\]
\[
= 36 - 60 < 0 \quad \Rightarrow \text{no roots}
\]
11. no roots

12. (1 point) Find the roots of $x^2 - 4x - 10$
\[
-b + \sqrt{b^2 - 4ac}
\]
\[
2a
\]
\[
= (-4) + \sqrt{(-4)^2 - 4(1)(-10)}
\]
\[
= 2 \cdot 2 = 4 + 2\sqrt{19}
\]
\[
= 2 + \sqrt{19}
\]
12. $2 + \sqrt{19} \leq 2 - \sqrt{19}$

13. (1 point) Find a root of $6x^3 + 12x^2 + 9x + 3$

Factors of $3$: $1, 3, -1, -3$

$x = 1$: $6(1)^3 + 12(1)^2 + 9(1) + 3 \neq 0$

$x = 3$: $6(3)^3 + 12(3)^2 + 9(3) + 3 \neq 0$

$x = -1$: $6(-1)^3 + 12(-1)^2 + 9(-1) + 3$
\[
= -6 + 12 - 9 + 3
\]
\[
= 0
\]
13. $-1$
14. (1 point) Completely factor \(3x^3 - 4x^2 + x - 10\). (Hint: 2 is a root.) (Your answer should be a product of a constant polynomial (a number), and perhaps some monic linear polynomials (monic i.e. the leading coefficient equals to 1) and monic quadratic polynomials (which have no roots). 

\[
\begin{array}{c|cccc}
& 3 & -4 & 1 & -10 \\
\hline
2 & & & & \\
\hline
& 6 & 2 & 5 & 0
\end{array}
\]

\[
p(x) = (x-2)(3x^2 + 3x + 5)
\]

15. (1 point) Completely factor \(2x^3 + 9x^2 + 7x - 6\). (Hint: -2 is a root.) (Your answer should have the same form as described in #21.) 

\[
\begin{array}{c|cccc}
& 2 & 9 & 7 & -6 \\
\hline
-2 & & & & \\
\hline
& -4 & -10 & 6
\end{array}
\]

\[
p(x) = (x+2)(2x^2 + 5x - 3)
\]
Graphs

16. (1 point) List all of the monic linear factors of $p(x)$ that you know of from the graph below.

Roots are $-5, -2, 0$ (that we know about)

$\rightarrow$ Monic linear factors that we know about are:

$x - (-5), x - (-2), x - 0$
17. (1 point) Graph $3\sqrt{x} + 1$, and label its $x$- and $y$-intercepts.

$\sqrt{x} + 1 = 0 \rightarrow x = -1$

$y$-intercept: $x = 0 \rightarrow y = 3\sqrt{0} + 1 \rightarrow y = 3$

$(-1, 0)$, $(0, 3)$

18. (1 point) Graph $-4x - 2$, and label its $x$- and $y$-intercepts.

$x = 0 \rightarrow y = -2$

$y = 0 \rightarrow -4x - 2 = 0 \rightarrow -4x = 2 \rightarrow x = -\frac{1}{2}$

$(-\frac{1}{2}, 0)$, $(0, -2)$

Points earned: _______ out of a possible 2 points
19. (1 point) Graph \( \sqrt[3]{\frac{x}{3}} - 1 \), and label its x- and y-intercepts.

\[
y = 0 \rightarrow 3\sqrt[3]{\frac{x}{3}} - 1 = 0
\]
\[
\frac{x}{3} = 1
\]
\[
x = 3(1)^3
\]
\[
x = 3
\]

20. (1 point) Graph \(-3(x - 1)^2 - 1\); label its vertex, and y-intercept. (It does not have an x-intercept.)

Vertex \( = (1, -1) \)
\[
x = 0 \rightarrow y = -3(0 - 1)^2 - 1
\]
\[
= -3(-1)^2 - 1
\]
\[
= -3(1) - 1
\]
\[
= -3 - 1
\]
\[
= -4
\]