# Final Prep : Early material

① What is an arithmetic sequence?

$a_1, a_2, a_3, \ldots$ $\qquad a_{n+1} = a_n + d$

($d \in \mathbb{R}$ is constant)

② What is a geometric sequence?

$a_{n+1} = r a_n$ $\qquad$ ($r \in \mathbb{R}$ is constant)

③ What's the $170^{th}$ term of $3, 5, 7, 9, \ldots$?

$+2 \quad +2 \quad +2$

$a_n = a_1 + (n-1)d$

$\begin{pmatrix} n = 170 \\ d = 2 \end{pmatrix}$ $\qquad a_{170} = 3 + 169(2) = 3 + 338 = \underline{341}$

④ What's the $214^{th}$ term of $-5, 15, -45, 135, \ldots$?

$\times(-3) \quad \times(-3) \quad \times(-3)$

$a_n = a_1 (r^{n-1})$

$\begin{pmatrix} n = 214 \\ r = -3 \end{pmatrix}$ $\qquad a_{214} = (-5)(-3)^{213}$

⑤ For which kinds of sequences do you know how to find the sum of the first k terms?

arithmetic

#8 Sum $= \dfrac{7/5}{1 - 1/5} = \dfrac{7/5}{4/5}$

$= \dfrac{7}{\cancel{5}}\left(\dfrac{\cancel{5}}{4}\right) = 7/4$

⑥ For which kinds of sequences have we learned how to find the sum of <u>all</u> the terms?

geometric, when $-1 < r < 1$

⑦ What's the sum of the first 40 terms of $-7, -4, -12, \ldots$?

$+3 \quad +3$

Sum of the first k terms

$= \dfrac{k(a_1 + a_k)}{2}$ $\qquad \begin{pmatrix} k = 40 \\ a_1 = -7 \end{pmatrix}$

$= \dfrac{40}{2}\left[-7 + (-7 + \underbrace{39(3)}_{40(3) = 120})\right]$

$= 20(-14 + 117)$ $\qquad -3 \rightarrow 117$

$= 20(103) = 2{,}060$

⑧ What's $\displaystyle\sum_{i=1}^{\infty} \left(\frac{7}{5^i}\right)$?

$= \dfrac{a_1}{1 - r}$ $\qquad \begin{pmatrix} a_1 = 7/5 \\ r = 1/5 \end{pmatrix}$ $\qquad \dfrac{7}{5^i} = 7\left(\dfrac{1}{5}\right)^i$

(see answer under #5)

⑨ Find $\sum_{i=1}^{3} (1-i^2) = (1-(1)^2) + (1-(2)^2) + (1-(3)^2)$

$= 0 + (-3) + (-8)$

$= -11$

⑩ Find $\sum_{i=1}^{30} 5$.

$= \underbrace{5+5+5+\cdots+5}_{30} = 5(30) = 150$

⑪ How many ways are there to order a set of 48 objects?

$48! = 48(47)(46)\cdots(3)(2)(1)$

"48 factorial"

→ ⑫ How many ways can you <u>choose</u> and then <u>order</u> 17 objects from a set of 58 objects?

$\frac{58!}{(58-17)!} = \frac{58!}{41!}$

→ ⑬ How many subsets of a set of 98 objects contain exactly 23 objects?

$\frac{98!}{(98-23)!\,23!} = \frac{98!}{75!\,23!} = \binom{98}{23}$ → "binomial coefficient"

⑭ What does "options multiply" mean?

To find the number of ways to make a sequence of decisions: Count how many ways each decision can be made, and then multiply all these numbers together.

⑮ Write $\binom{7}{4}$ as a natural number in standard form.

$\binom{7}{4} = \frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\,4!} = \frac{7(6)(5)\cancel{(4!)}}{3!\,\cancel{4!}}$

$= \frac{7\cancel{(6)}(5)}{\cancel{3(2)}(1)} = 7(5) = 35$

Function composition

(16) $f(x)=x^2+2$, $g(x)=3x-1$. Find $f\circ g(x)$ and $g\circ f(x)$.

$f\circ g(x) = f(g(x)) = f(3x-1)$

"f composed with g" $= (3x-1)^2+2$

$g\circ f(x) = g(f(x))$
$= g(x^2+2)$
$= 3(x^2+2)-1$
$= 3x^2+6-1$
$= 3x^2+5$

(17) $f(x)=2(x-4)^3+1$. Find $f^{-1}(y)$.

$y = 2(x-4)^3+1$

→ Solve for $x$

$y-1 = 2(x-4)^3$

$\frac{y-1}{2} = (x-4)^3$

$\sqrt[3]{\frac{y-1}{2}} = x-4$

$x = 4+\sqrt[3]{\frac{y-1}{2}}$

Answer

$f^{-1}(y) = 4+\sqrt[3]{\frac{y-1}{2}}$

$f^{-1}(x) = 4+\sqrt[3]{\frac{x-1}{2}}$

(18) What are the implied domains of the following functions:

→ The subset of $\mathbb{R}$ for which the function is defined.

$f(x)=x$ $\mathbb{R}$

$f(x)=x^2$ $\mathbb{R}$

$f(x)=x^3$ $\mathbb{R}$

$f(x)=4$ $\mathbb{R}$

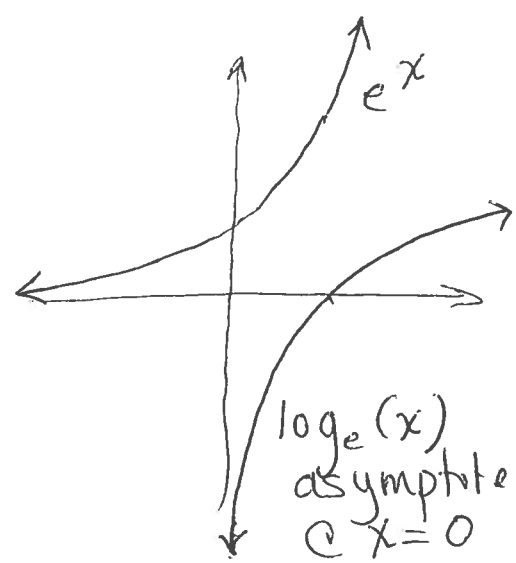
$f(x)=\sqrt[2]{x}$ $x\geq 0$ Implied domain $=[0,\infty)$

$f(x)=\sqrt[3]{x}$ $\mathbb{R}$

$f(x)=1/x$ $\mathbb{R}-\{0\}$

$f(x)=e^x$ $\mathbb{R}$

$f(x)=\log_e(x)$ $(0,\infty)$



(19) What's the implied domain of $f(x) = \frac{27}{3}x^5 - 3x^2 + 27$?

$\mathbb{R}$ (a polynomial is defined for every $x \in \mathbb{R}$)

(20) What's the implied domain of $r(x) = \frac{3x-7}{x^2-4}$?

Not defined when
$x^2 - 4 = 0$
$\rightarrow x = 2$ or $-2$

Implied domain $= \mathbb{R} - \{2, -2\}$

(21) What's the implied domain of $g(x) = \frac{1}{e^x}$?

Not defined if
$e^x = 0$
However, $e^x$ never equals 0, $\rightarrow$ no matter what $x$ you plug in.

$\Longrightarrow$ Implied domain $= \mathbb{R}$

(22) What's the implied domain of $\sqrt[2]{(7-x)}$?

$7 - x \geq 0$
$7 \geq x$
$x \leq 7$

$\hookrightarrow = (-\infty, 7]$

(23) What's the implied domain of $5x^2 - \sqrt[3]{2x-3}$?

This function is well-defined for any $x \in \mathbb{R}$ ... nothing $\rightarrow$ would cause division by zero, square root or logarithm of a negative #, etc.

$\longrightarrow$ Implied domain $= \mathbb{R}$

(24) What's the implied domain of $2x - \log_e(3x+4)$?

$3x + 4 > 0$
$3x > -4$
$x > -4/3$

$\hookrightarrow = \left(\frac{-4}{3}, \infty\right)$

(25) What's the implied domain of $e^{\sqrt[3]{5x-2}} + 3x^2 - 5$?

Same as (23): no issues like dividing by zero, square root of a negative number, etc.

Implied domain $= \mathbb{R}$