Math 1050–4 Final Exam (practice) #3

April 17, 2015

- No notes, books, etc. are allowed.
- Answer the questions in the spaces provided.
- You have 2 hours to complete this exam.
- **Good luck!**

Name: _____ uNID: _____

Page	Points	Score
2	6	
3	6	
4	7	
5	9	
6	5	
7	7	
8	8	
9	3	
10	12	
11	12	
12	6	
13	4	
14	4	
Total:	89	

Points earned: _____
out of a possible 0 points

1. (1 point) Find $\sum_{i=1}^{4}(i^2-1)$

$= (1^2-1) + (2^2-1) + (3^2-1) + (4^2-1)$

$= (1-1) + (4-1) + (9-1) + (16-1)$

$= 0 + 3 + 8 + 15$

$= 11 + 15 = 26$

1. 26

2. (1 point) Find $\sum_{i=1}^{\infty}\frac{2}{6^i}$

$\frac{2}{6^i} = 2\left(\frac{1}{6}\right)^i \longrightarrow a_1 = \frac{2}{6} = \frac{1}{3}$

$r = 1/6$

$= \frac{a_1}{1-r}$

$= \frac{1/3}{1-1/6} = \frac{1/3}{5/6} = \frac{1}{\cancel{3}}\left(\frac{\cancel{6}^{2}}{5}\right) = \frac{2}{5}$

2. $2/5$

3. (1 point) What is the 61-st term of the sequence 7, 11, 15, 19, . . .?

$\longrightarrow$ arithmetic sequence with $d = 4$ $(a_1 = 7)$

$a_n = a_1 + (n-1)d$

$a_{61} = 7 + (61-1)4$

$= 7 + 60(4)$

$= 7 + 240 = 247$

3. 247

4. (1 point) What is the 57-th term of −3, 6, −12, 24, . . .?

• Geometric sequence with $r = \frac{6}{-3} = -2$

$a_n = a_1(r^{n-1})$

$a_{57} = (-3)(-2)^{57-1}$

4. $(-3)(-2)^{56}$

5. (2 points) What is the sum of the first 60 terms of the sequence 3, 5, 7, 9, . . .?

• Arithmetic sequence with $d = 2$

$\text{Sum} = \frac{1}{2}k(a_1 + a_k)$

$= \frac{1}{2}(60)(3 + (3 + (60-1)2))$

$= 30(3 + 3 + 59(2))$

$= 30(6 + 118)$

$= 30(124)$

5. $30(124)$

Points earned: _____
out of a possible 6 points

6. (1 point) Suppose a set A contains 243 objects. How many 92 object subsets of A are there?

"Subsets" $\longrightarrow$ order does not matter

Answer: (binomial coefficient) $\binom{243}{92}$ or $\frac{243!}{(243-92)!\,92!}$ or $\frac{243!}{151!\,92!}$

6. _____

7. (1 point) How many ways are there to choose and order 49 objects from a collection of 304 objects?

Answer: $\frac{304!}{(304-49)!}$ or $\frac{304!}{255!}$

7. _____

8. (1 point) How many different ways are there to order 93 different objects?

8. $93!$

9. (1 point) You're decorating a room by choosing a color of paint for the walls, and a color of carpet. You have 6 different colors of paint to choose from, and 11 different colors of carpet to choose from. How many different wall and floor color combinations could you create?

Options Multiply: 6×11

9. 66

10. (2 points) Write $\binom{9}{3}$ as an integer in standard form.

$$\frac{9!}{(9-3)!\,3!} = \frac{9!}{6!\,3!} = \frac{\overset{3}{\cancel{9}}\overset{4}{\cancel{(8)}}(7)\cancel{(6!)}}{\cancel{6!}\cancel{(3)}\cancel{(2)}(1)}$$

$= 3(4)(7)$

$= 12(7)$

$= 84$

10. 84

Points earned: _____
out of a possible 6 points

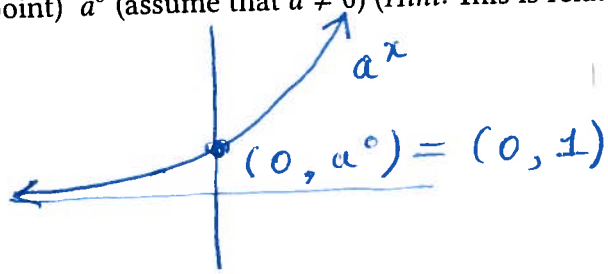
For #11–#15, write the given number as a rational number in standard form

11. (1 point) $\left(\frac{27}{8}\right)^{-\frac{2}{3}} = \frac{1}{\left(\frac{27}{8}\right)^{2/3}} = \frac{1}{\left(\sqrt[3]{27/8}\right)^2} = \frac{1}{\frac{(\sqrt[3]{27})^2}{(\sqrt[3]{8})^2}}$

$= \frac{(\sqrt[3]{8})^2}{(\sqrt[3]{27})^2} = \frac{(2)^2}{(3)^2} = \frac{4}{9}$

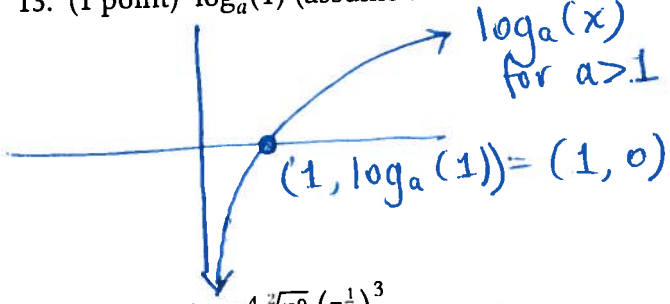
11. 4/9

12. (1 point) a^0 (assume that $a \neq 0$) (*Hint:* This is related to the graph of a^x.)



12. 1

13. (1 point) $\log_a(1)$ (assume that $a > 0$ and $a \neq 1$) (*Hint:* This is related to the graph of $\log_a(x)$.)



13. 0

14. (2 points) $7^{-4}\sqrt[2]{7^9}\left(7^{\frac{1}{2}}\right)^3$

$7^{-4}(7^9)^{1/2}\, 7^{\frac{1}{2}(3)}$

$= 7^{-4}\, 7^{9(1/2)}\, 7^{3/2}$

$= 7^{-4+9/2+3/2}$

$= 7^{-4+12/2}$

$= 7^{-4+6}$

$= 7^2$

14. 49

15. (2 points) $\log_3\left(\sqrt[5]{\frac{1}{81}}\right)$

$\log_3\left(\sqrt[5]{81^{-1}}\right)$

$= \log_3\left(\sqrt[5]{(3^4)^{-1}}\right)$

$= \log_3\left(\sqrt[5]{3^{-4}}\right)$

$= \log_3\left((3^{-4})^{1/5}\right)$

$= \log_3\left(3^{-4(1/5)}\right)$

$= \log_3\left(3^{-4/5}\right)$

15. −4/5

Points earned: _____
out of a possible 7 points

16. (1 point) Find $g \circ f(x)$ if $f(x) = x + 2$ and $g(x) = x^2$

$$g \circ f(x) = g(f(x))$$
$$= g(x+2)$$
$$= (x+2)^2$$

16. $g \circ f(x) = (x+2)^2$

17. (2 points) Find x where $x^3\left(\frac{1}{2}x + 3\right)^3 = 8$

$$\left(x\left(\tfrac{1}{2}x+3\right)\right)^3 = 8$$
$$x\left(\tfrac{1}{2}x+3\right) = \sqrt[3]{8}$$
$$\tfrac{1}{2}x^2 + 3x = 2$$
$$\tfrac{1}{2}x^2 + 3x - 2 = 0$$
$$x = \frac{-3 \pm \sqrt{3^2 + 4(\frac{1}{2})(+2)}}{2(\frac{1}{2})}$$
$$x = \frac{-3 \pm \sqrt{9+4}}{1}$$

17. _____

#17 $x = -3 + \sqrt{13}$ & $x = -3 - \sqrt{13}$

18. (2 points) Find x where $2\left(\frac{e^{2x}}{e^{x+3}}\right) + 5 = 7$

$$2(e^{2x-(x+3)}) + 5 = 7$$
$$2(e^{2x-x-3}) = 7 - 5$$
$$e^{x-3} = \frac{7-5}{2}$$
$$e^{x-3} = \frac{2}{2} = 1$$
$$\rightarrow x - 3 = \log_e(1) = 0$$
$$\rightarrow x = 3$$

18. $x = 3$

19. (2 points) Find x where $4\log_e(x) + \log_e(x^3) + 8 = 11$

$$\log_e(x^4) + \log_e(x^3) + 8 = 11$$
$$\log_e(x^4(x^3)) + 8 = 11$$
$$\log_e(x^7) = 11 - 8$$
$$\log_e(x^7) = 3$$
$$x^7 = e^3$$
$$x = (e^3)^{1/7}$$
$$x = e^{3/7}$$

19. $x = e^{3/7}$

20. (2 points) Find the inverse of $g(x) = 7\log_e(x+3)$

$$y = 7\log_e(x+3)$$
$$\frac{y}{7} = \log_e(x+3)$$
$$e^{y/7} = x + 3$$
$$x = e^{y/7} - 3$$
$$f^{-1}(y) = e^{y/7} - 3$$

20. _____

Points earned: _____
out of a possible 9 points

21. (1 point) What is the implied domain of $g(x) = \frac{x^3}{2} - 7\sqrt[3]{x-4}$?

21. $\mathbb{R}$

22. (2 points) What is the implied domain of $f(x) = x^2 - 2x + \log_e(3 - 7x)$?

$3 - 7x > 0$

$3 > 7x$

$3/7 > x$

$x < 3/7$

$(-\infty, 3/7)$

22. $(-\infty, 3/7)$

23. (2 points) Find $\frac{x^3 - 3x^2 - 5x + 14}{x^2 - 4}$

$$
\begin{array}{r|l}
 & x \quad -3 \\
\hline
x^2 - 4 & x^3 - 3x^2 - 5x + 14 \\
 & -(x^3 \qquad\quad -4x \qquad\quad) \\
\hline
 & -3x^2 - x + 14 \\
 & -(-3x^2 \qquad\quad + 12) \\
\hline
 & -x + 2
\end{array}
$$

← remainder

23. $x - 3 + \frac{-x+2}{x^2-4}$

Points earned: _____
out of a possible 5 points

24. (1 point) Find a root of $x^3 + 2x^2 - x + 6$

Factors of 6: 1, 2, 3, 6, -1, -2, -3, -6

$x = 1 \longrightarrow 1^3 + 2(1)^2 - (1) + 6 = \cancel{1} + 2 - \cancel{1} + 6 \neq 0$

$x = 2, 3, 6$ would all be positive (i.e. $\neq 0$)

$x = -1 \longrightarrow (-1)^3 + 2(-1)^2 - (-1) + 6 = -\cancel{1} + 2 + \cancel{1} + 6 \neq 0$

$x = -2 \longrightarrow (-2)^3 + 2(-2)^2 - (-2) + 6 = -\cancel{8} + 2(\cancel{4}) + 2 + 6 \neq 0$

$x = -3 \longrightarrow (-3)^3 + 2(-3)^2 - (-3) + 6$

$= -27 + 2(9) + 3 + 6 = -27 + 18 + 9$

$= -27 + 27$

$= 0$

24. $x = -3$

25. (2 points) How many roots does $2x^2 - 3x + 4$ have?

$b^2 - 4ac$ = discriminat

If discriminant is $> 0 \longrightarrow$ 2 roots

$= 0 \longrightarrow$ 1 root

$< 0 \longrightarrow$ no roots

$b^2 - 4ac = (-3)^2 - 4(2)(4)$

$= 9 - 8(4) = 9 - 32 < 0$

25. zero roots

26. (2 points) Completely factor $-2x^3 + 2x + 12$. (*Hint:* 2 is a root.)

2	-2	0	2	12
		-4	-8	-12
	-2	-4	-6	0

$b^2 - 4ac = (-4)^2 - 4(-2)(-6)$

$= 16 - 8(6)$

$= 16 - 48 < 0$

$p(x) = (x-2)\underbrace{(-2x^2 - 4x - 6)}_{\text{how many roots? none}} = -2(x-2)(x^2 + 2x + 3)$

26. _____

27. (2 points) Complete the square: Write $-2x^2 - 4x - 5$ in the form $\alpha(x+\beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

$-2(x^2 + 2x) - 5$

$-2(x^2 + 2x + 1) - 5 - 1(-2)$

$-2(x+1)^2 - 5 + 2$

$-2(x+1)^2 - 3$

27. _____

Points earned: _____
out of a possible 7 points

28. (1 point) $|x - y|$ is the distance between which two numbers?

Note: $|x+y| = |x-(-y)|$

$\longrightarrow$ distance between x & $-y$.

28. x & y

29. (2 points) Solve for x if $|3x - 2| < 4$

$-4 < 3x - 2 < 4$

$-4 < 3x - 2$ & $3x - 2 < 4$

$-4 + 2 < 3x$

$(-4+2)/3 < x \longrightarrow \frac{-2}{3} < x$

$3x < 4 + 2$

$x < \frac{4+2}{3}$

$x < 6/3 \longrightarrow x < 2$

29. $\frac{-2}{3} < x < 2$

30. (1 point) What is the determinant of the matrix below?

$$\begin{pmatrix} 2 & -3 \\ 1 & -5 \end{pmatrix}$$

$$\det\begin{pmatrix} 2 & -3 \\ 1 & -5 \end{pmatrix} = 2(-5) - (-3)(1)$$
$$= -10 + 3$$
$$= -7$$

30. -7

31. (2 points) Find the product

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \quad 0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} & (1 \quad 0)\begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ (3 \quad 1)\begin{pmatrix} 1 \\ 0 \end{pmatrix} & (3 \quad 1)\begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

31.

32. (2 points) What is the inverse of the matrix below?

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}}\begin{pmatrix} 3 & -4 \\ -2 & 1 \end{pmatrix}$$

$$= \frac{1}{1(3) - 4(2)}\begin{pmatrix} 3 & -4 \\ -2 & 1 \end{pmatrix}$$

$$\frac{-1}{5}\begin{pmatrix} 3 & -4 \\ -2 & 1 \end{pmatrix}$$

32.

Points earned: _____
out of a possible 8 points

33. (1 point) Write the following system of three linear equations in three variables as a matrix equation

$$\begin{aligned} 2x - y + z &= 2 \\ y + 2z &= 1 \\ -x + y - z &= 0 \end{aligned}$$

$$\begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ -1 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

34. (2 points) Solve for x, y, and z if

$$\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}$$

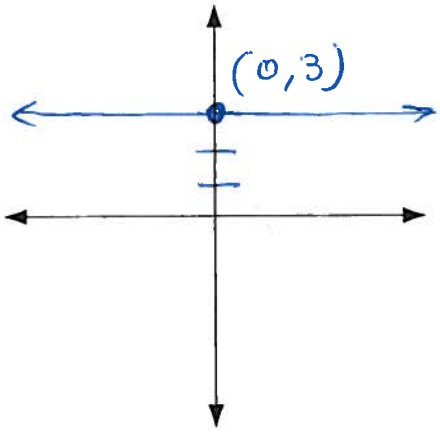
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 + (-2) + 0 \\ 2 + 4 + 1 \\ 8 + 10 + 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 9 \\ 20 \end{pmatrix}$$

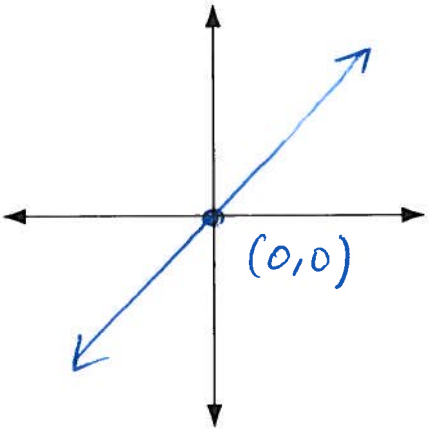
Points earned: _____
out of a possible 3 points

For #35–#40, mark and label x- and y-intercepts (if there are any).

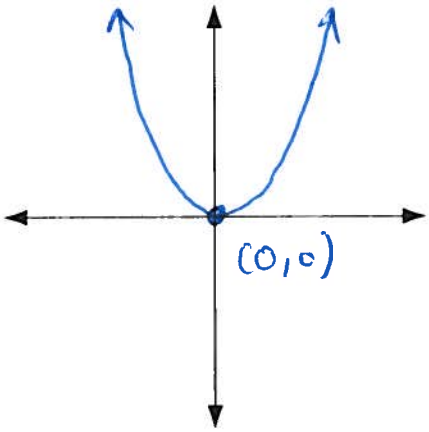
35. (2 points) 3



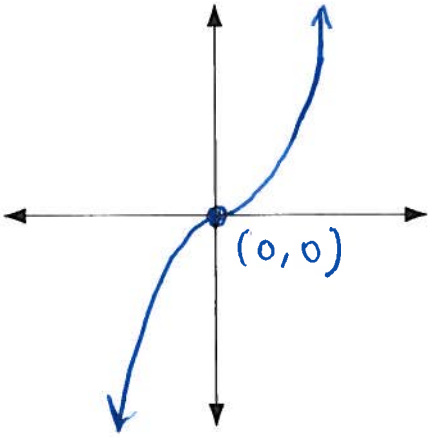
36. (2 points) x



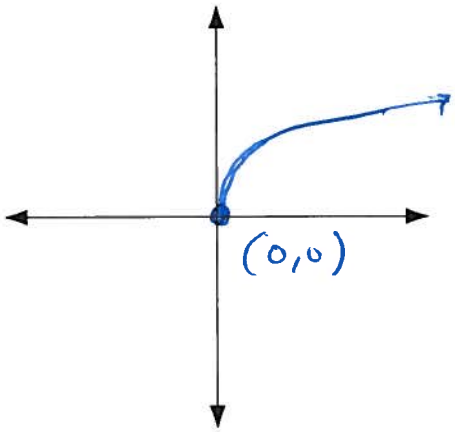
37. (2 points) x^2



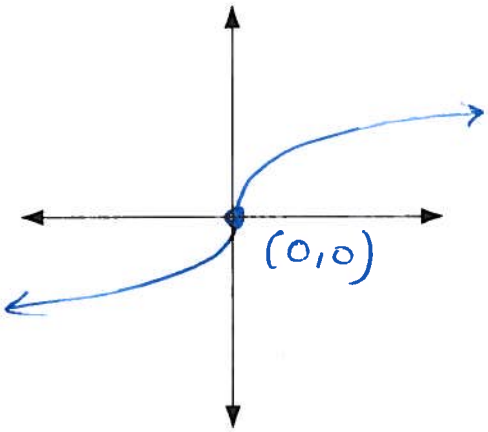
38. (2 points) x^3



39. (2 points) $\sqrt[3]{x}$



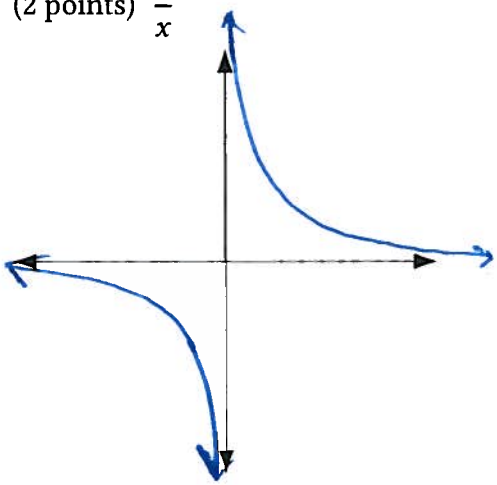
40. (2 points) $\sqrt[3]{x}$



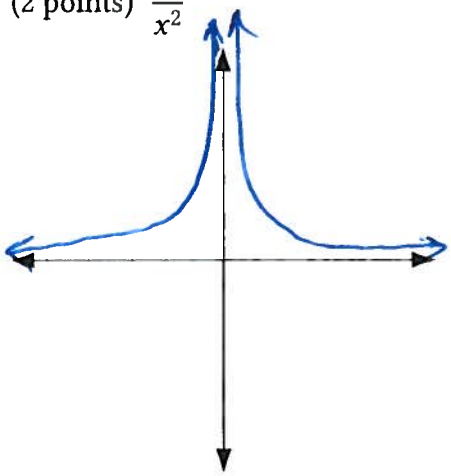
Points earned: _____
out of a possible 12 points

For #41–#46, mark and label x- and y-intercepts (if there are any).

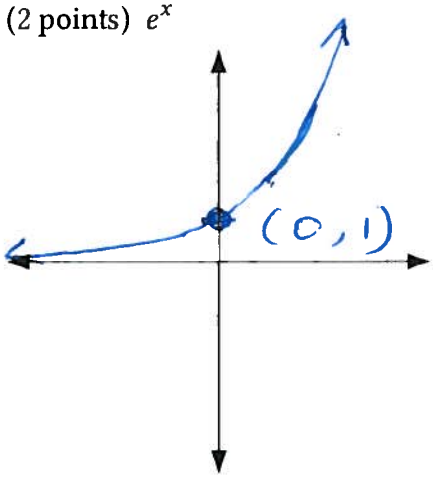
41. (2 points) $\frac{1}{x}$



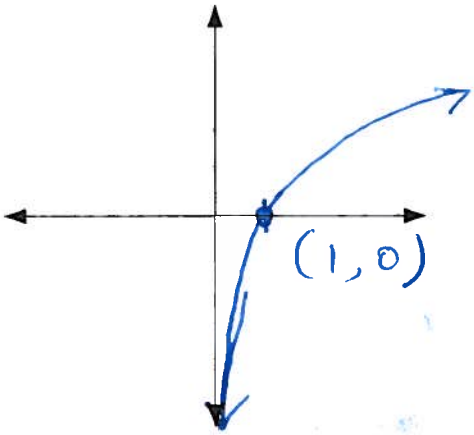
42. (2 points) $\frac{1}{x^2}$



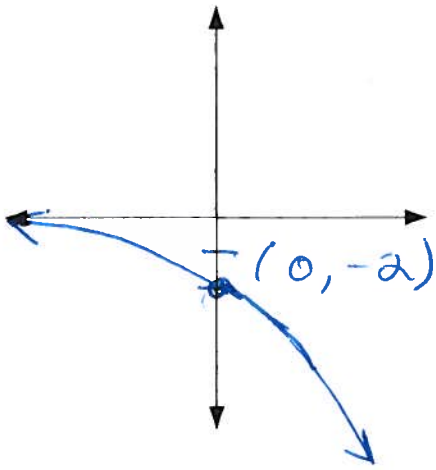
43. (2 points) e^x



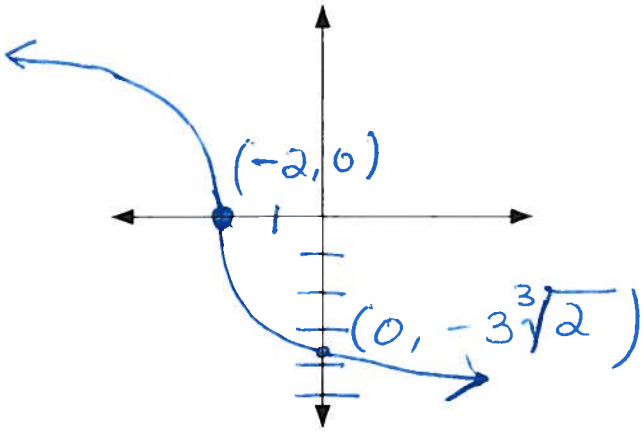
44. (2 points) $\log_e(x)$



45. (2 points) $-2e^{-x}$

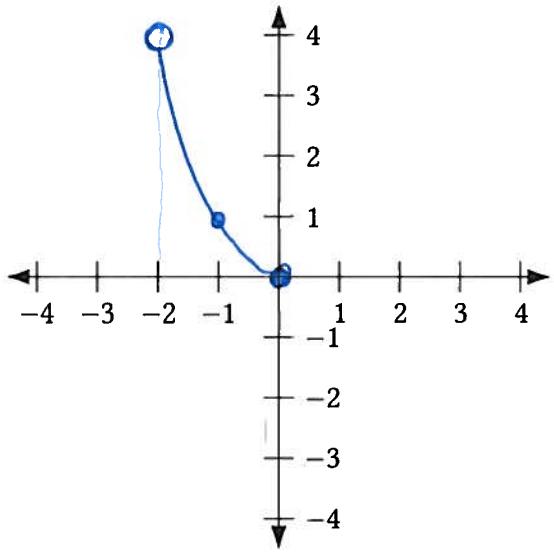


46. (2 points) $-3\sqrt[3]{x+2}$

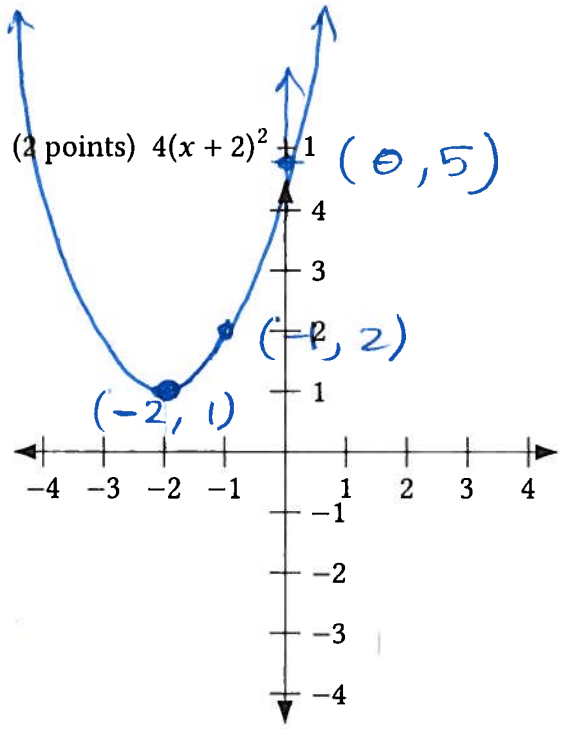


Points earned: _____
out of a possible 12 points

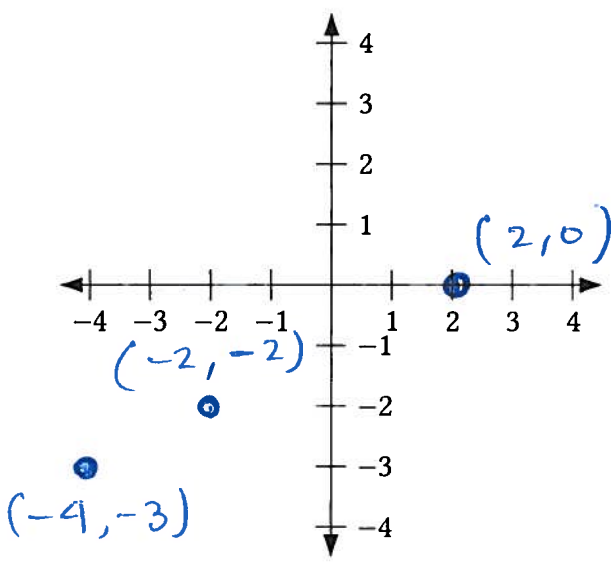
47. (2 points) $f : (-2, 0] \to \mathbb{R}$
where $f(x) = x^2$



49. (2 points) $4(x+2)^2 + 1$



48. (2 points) $g : \{-4, -2, 2\} \to \mathbb{R}$
where $g(x) = \frac{1}{2}x - 1$



$g(-4) = \frac{1}{2}(-4) - 1$
$= -2 - 1 = -3$

$g(-2) = \frac{1}{2}(-2) - 1$
$= -1 - 1 = -2$

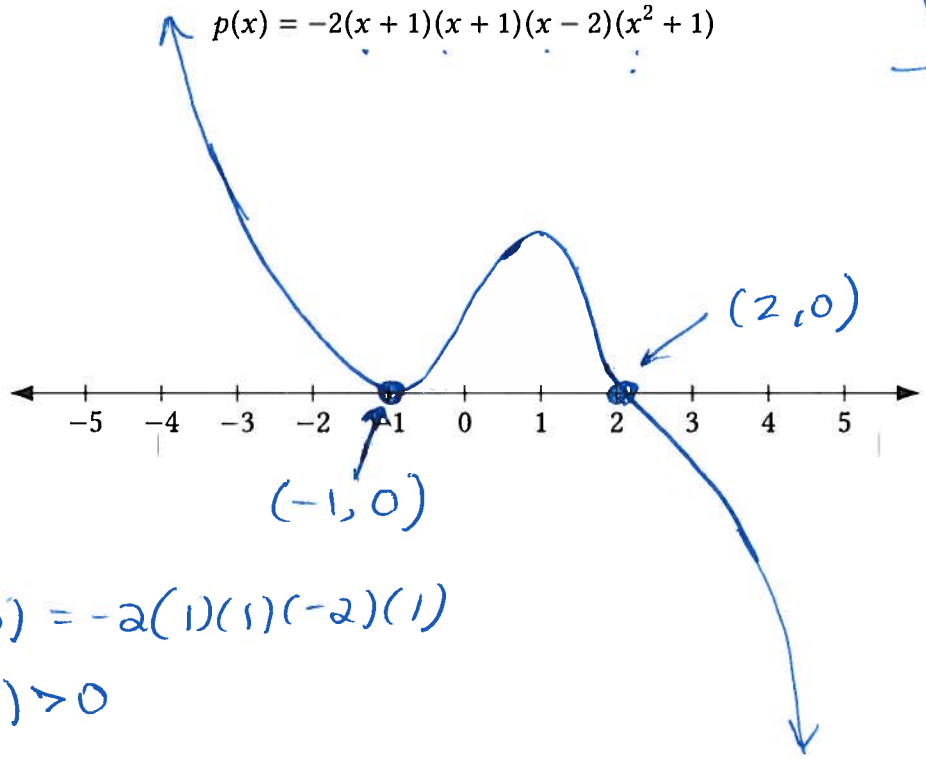
$g(2) = \frac{1}{2}(2) - 1$
$= 1 - 1 = 0$

Points earned: _____
out of a possible 6 points

Leading term $= -2x^5$

50. (2 points) Graph $p(x)$ and label all x-intercepts:

$$p(x) = -2(x+1)(x+1)(x-2)(x^2+1)$$



$x = 0$

$\rightarrow p(0) = -2(1)(1)(-2)(1)$

$\rightarrow p(0) > 0$

51. (2 points) Graph $r(x)$, and label all x-intercepts and all vertical asymptotes:

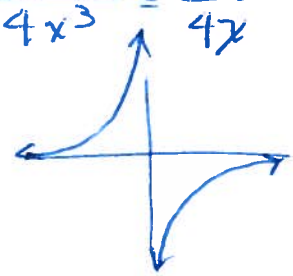
$$r(x) = \frac{-3(x-1)(x-1)}{4(x+2)(x^2+1)}$$

$x = 0$

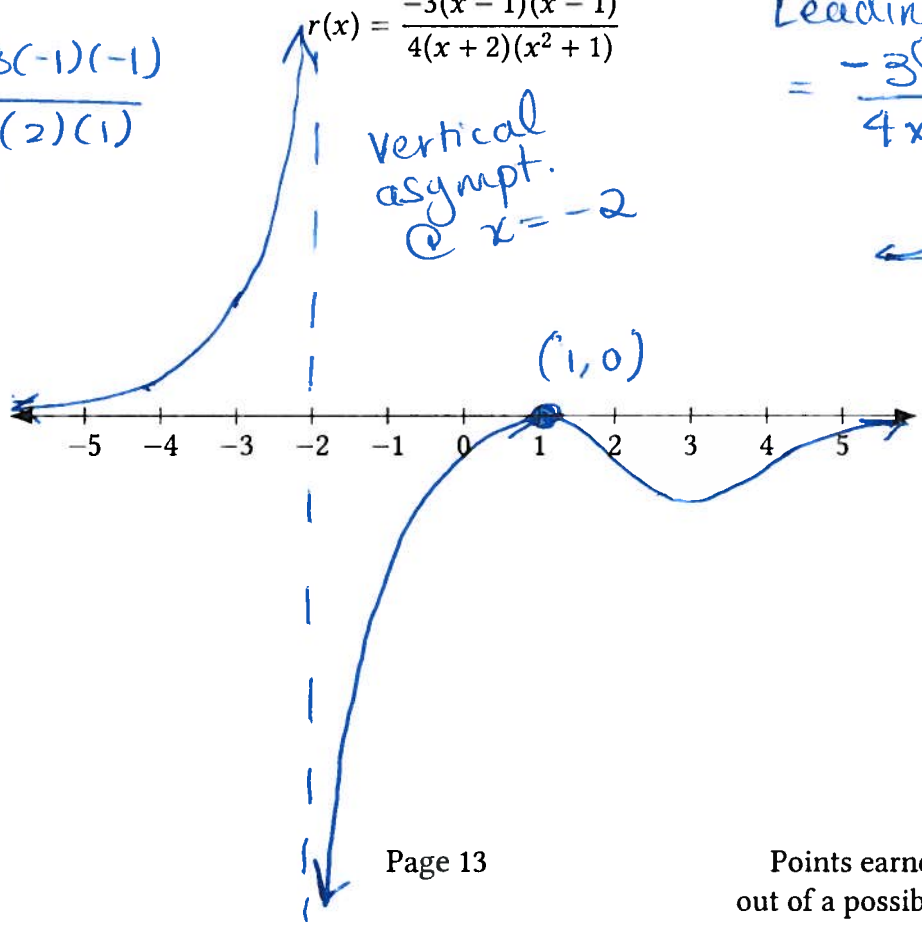
$\rightarrow r(0) = \dfrac{-3(-1)(-1)}{4(2)(1)}$

$\rightarrow r(0) < 0$

Leading term $= \dfrac{-3x^2}{4x^3} = \dfrac{-3}{4x}$

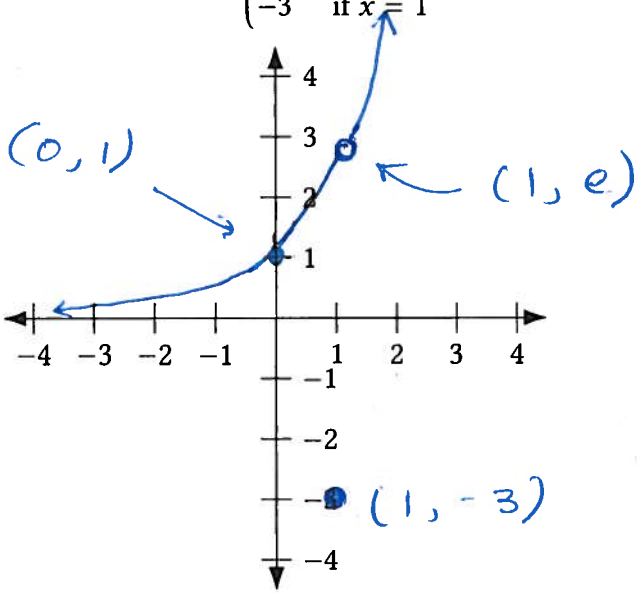


vertical asympt. @ $x = -2$

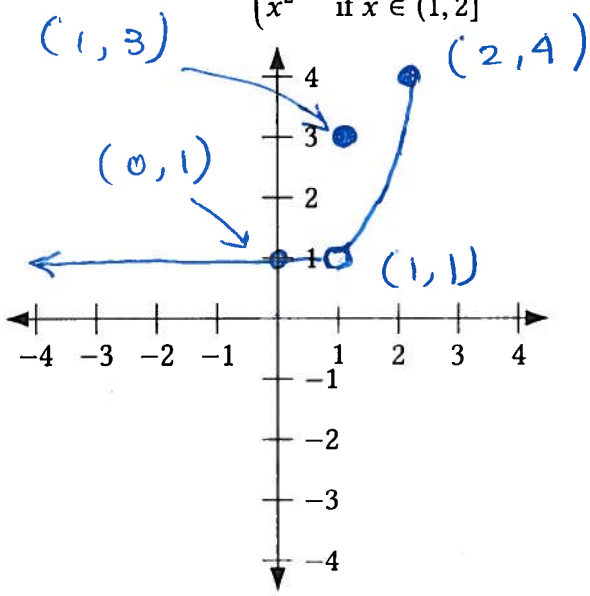


Points earned: _____
out of a possible 4 points

52. (2 points) $h(x) = \begin{cases} e^x & \text{if } x \neq 1 \\ -3 & \text{if } x = 1 \end{cases}$



53. (2 points) $m(x) = \begin{cases} 1 & \text{if } x \in (-\infty, 1) \\ 3 & \text{if } x = 1 \\ x^2 & \text{if } x \in (1, 2] \end{cases}$



Points earned: _____
out of a possible 4 points