Math 1050–4 Final Exam (practice) #3
April 17, 2015

- No notes, books, etc. are allowed.
- Answer the questions in the spaces provided.
- You have 2 hours to complete this exam.
- Good luck!

Name: ___________________________ uNID: ________________________________

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Points earned: ______ out of a possible 0 points
1. (1 point) Find \( \sum_{i=1}^{4} (i^2 - 1) \)

\[
= (1^2 - 1) + (2^2 - 1) + (3^2 - 1) + (4^2 - 1)
= (1 - 1) + (4 - 1) + (9 - 1) + (16 - 1)
= 0 + 3 + 8 + 15
= 26
\]

2. (1 point) Find \( \sum_{i=1}^{6} \frac{2}{6^i} \)

\[
= \frac{\frac{2}{6}}{1 - \frac{1}{6}}
= \frac{\frac{2}{6}}{\frac{5}{6}}
= \frac{2}{5}
\]

3. (1 point) What is the 61-st term of the sequence 7, 11, 15, 19, ...?

\[
\alpha_n = \alpha_1 + (n-1)d
\]

\[
\alpha_{61} = 7 + (61 - 1)4
= 7 + 60 \cdot 4
= 7 + 240
= 247
\]

4. (1 point) What is the 57-th term of -3, 6, -12, 24, ...?

\[
\text{Geometric sequence with } r = \frac{-2}{3} = -2
\]

\[
\alpha_n = \alpha_1 \cdot r^{n-1}
\]

\[
\alpha_{57} = (-3) \cdot (-2)^{57-1}
= 30 \cdot 124
= 3720
\]

5. (2 points) What is the sum of the first 60 terms of the sequence 3, 5, 7, 9, ...?

\[
\text{Arithmetic sequence with } d = 2
\]

\[
\text{Sum} = \frac{1}{2} \cdot k \cdot (\alpha_1 + \alpha_k)
= \frac{1}{2} (60)(3 + (3 + (60 - 1)2))
= 30(3 + 3 + 59(2))
= 30(6 + 118)
= 30(124)
\]

Points earned: _______ out of a possible 6 points
6. (1 point) Suppose a set \( A \) contains 243 objects. How many 92 object subsets of \( A \) are there?

"Subsets" \[ \binom{243}{92} \] \[
\frac{243!}{(243-92)! 92!} \]

or \[
\frac{243!}{151! 92!} \]

7. (1 point) How many ways are there to choose and order 49 objects from a collection of 304 objects?

Answer: \[
\frac{304!}{(304-49)!} \] \[
\frac{304!}{255!} \]

8. (1 point) How many different ways are there to order 93 different objects?

9. (1 point) You’re decorating a room by choosing a color of paint for the walls, and a color of carpet. You have 6 different colors of paint to choose from, and 11 different colors of carpet to choose from. How many different wall and floor color combinations could you create?

Options: Multiply: \[
6 \times 11 \]

10. (2 points) Write \( \binom{9}{3} \) as an integer in standard form.

\[
\frac{9!}{(9-3)! 3!} = \frac{9!}{6! 3!} = \frac{3 \cdot 4 \cdot 5 \cdot 6}{3 \cdot 2 \cdot 1} = 3 \cdot 4 \cdot 5 = 12 \cdot 5 = 84
\]

Points earned: \[\_\_\_\_\_\_\_] out of a possible 6 points
For #11–#15, write the given number as a rational number in standard form

11. (1 point) \( \left( \frac{27}{8} \right)^{\frac{2}{3}} = \left( \frac{27}{8} \right)^{\frac{2}{3}} = \left( \frac{3\sqrt[3]{9}}{8} \right)^2 = \left( \frac{\sqrt[3]{8}}{\sqrt[3]{2} \sqrt[3]{7}} \right)^2 = \frac{4}{9} \)

12. (1 point) \( a^0 \) (assume that \( a \neq 0 \)) (Hint: This is related to the graph of \( a^x \).)

13. (1 point) \( \log_a(1) \) (assume that \( a > 0 \) and \( a \neq 1 \)) (Hint: This is related to the graph of \( \log_a(x) \).)

14. (2 points) \( 7^{-4} \sqrt[3]{7^2} \cdot (7^2)^3 \)

\[
14. 49
\]

15. (2 points) \( \log_3 \left( \sqrt[3]{\frac{1}{81}} \right) \)

\[
15. -\frac{4}{5}
\]
16. (1 point) Find \( g \circ f(x) \) if \( f(x) = x + 2 \) and \( g(x) = x^2 \)

\[
\begin{align*}
  g \circ f(x) &= g(f(x)) \\
  &= g(x + 2) \\
  &= (x + 2)^2
\end{align*}
\]

17. (2 points) Find \( x \) where \( x^3 \left( \frac{1}{2} x + 3 \right)^3 = 8 \)

\[
\begin{align*}
  \left( x \left( \frac{1}{2} x + 3 \right) \right)^3 &= 8 \\
  x \left( \frac{1}{2} x + 3 \right) &= \sqrt[3]{8} \\
  \frac{1}{2} x^2 + 3x &= 2
\end{align*}
\]

\[
\begin{align*}
  \frac{1}{2} x^2 + 3x - 2 &= 0 \\
  x &= \frac{-3 \pm \sqrt{9 - 4 \left( \frac{1}{2} \right)(-2)}}{2} \\
  x &= \frac{-3 \pm \sqrt{9 + 4}}{2} \\
  x &= \frac{-3 \pm 5}{2}
\end{align*}
\]

\( x = -3 + \sqrt{13} \) or \( x = -3 - \sqrt{13} \)

18. (2 points) Find \( x \) where \( 2 \left( \frac{e^{2x}}{e^x + 3} \right) + 5 = 7 \)

\[
\begin{align*}
  2 \left( \frac{e^{2x}}{e^x + 3} \right) + 5 &= 7 \\
  2 \left( \frac{e^{2x} - e^x - 3}{e^x + 3} \right) &= 2 \\
  e^x - 3 &= \frac{7 - 5}{2}
\end{align*}
\]

\( e^x - 3 = \frac{2}{2} = 1 \)

\( x - 3 = \log_e (1) = 0 \)

\( x = 3 \)

19. (2 points) Find \( x \) where \( 4 \log_e (x) + \log_e (x^3) + 8 = 11 \)

\[
\begin{align*}
  \log_e (x^4) + \log_e (x^3) + 8 &= 11 \\
  \log_e (x^7) &= 11 - 8 \\
  \log_e (x^7) &= 3
\end{align*}
\]

\( x^7 = e^3 \)

\( x = (e^3)^{1/7} \)

\( x = e^{3/7} \)

20. (2 points) Find the inverse of \( g(x) = 7 \log_e (x + 3) \)

\[
\begin{align*}
  y &= 7 \log_e (x + 3) \\
  \frac{y}{7} &= \log_e (x + 3) \\
  e^{y/7} &= x + 3
\end{align*}
\]

\( f^{-1}(y) = e^{y/7} - 3 \)
21. (1 point) What is the implied domain of \( g(x) = \frac{x^3}{2} - 7\sqrt{x-4} \)?

22. (2 points) What is the implied domain of \( f(x) = x^2 - 2x + \log_e(3 - 7x) \)?

\[
\begin{align*}
3 - 7x &> 0 \\
3 &> 7x \\
\frac{3}{7} &> x \\
x &< \frac{3}{7} \\
(-\infty, \frac{3}{7})
\end{align*}
\]

23. (2 points) Find \[
\frac{x^3 - 3x^2 - 5x + 14}{x^2 - 4}
\]

\[
\begin{align*}
x &-3 \\
x^2 - 4 &\left|\begin{array}{c}x^3 - 3x^2 - 5x + 14 \\
-x^3 + 4x \\
-3x^2 - x + 14 \\
-x^2 + 12 \\
-x + 2
\end{array}\right|
\text{remainder}
\end{align*}
\]

\[
x - 3 + \frac{-x + 2}{x^2 - 4}
\]

Points earned: _______ out of a possible 5 points
24. (1 point) Find a root of \( x^3 + 2x^2 - x + 6 \)

Factors of 6: 1, 2, 3, 6, -1, -2, -3, -6

\( x = 1 \rightarrow 1^3 + 2(1)^2 - (1) + 6 = 1 + 2 - 1 + 6 \neq 0 \)

\( x = 2, 3, 6 \) would all be positive (i.e. \( \neq 0 \))

\( x = -1 \rightarrow (-1)^3 + 2(-1)^2 - (-1) + 6 = -1 + 2 + 1 + 6 \neq 0 \)

\( x = -2 \rightarrow (-2)^3 + 2(-2)^2 - (-2) + 6 = -8 + 8 + 2 + 6 \neq 0 \)

\( x = -3 \rightarrow (-3)^3 + 2(-3)^2 - (-3) + 6 \)

\( = -27 + 18 + 9 = -27 + 27 = 0 \)

25. (2 points) How many roots does \( 2x^2 - 3x + 4 \) have?

\[
b^2 - 4ac = \text{discriminant}
\]

If discriminant is \( > 0 \) \( \rightarrow \) 2 roots

\( = 0 \) \( \rightarrow \) 1 root

\( < 0 \) \( \rightarrow \) no roots

\[
b^2 - 4ac = (3)^2 - 4(2)(4) = 9 - 32 < 0
\]

26. (2 points) Completely factor \(-2x^3 + 2x + 12\). (Hint: 2 is a root.)

\[
b^2 - 4ac = (-4)^2 - 4(-2)(-6) = 16 - 48 < 0
\]

\[
(x - 2)(-2x^2 - 4x - 6) = -2 (x - 2)(x^2 + 2x + 3)
\]

27. (2 points) Complete the square: Write \(-2x^2 - 4x - 5\) in the form \(a(x + \beta)^2 + \gamma\) where \(a, \beta, \gamma \in \mathbb{R}\).

\[
-2(x^2 + 2x) - 5
-2(x^2 + 2x + 1) - 5 - 1(-2)
-2(x + 1)^2 - 3
\]
28. (1 point) \(|x - y|\) is the distance between which two numbers?

\[
\text{Note: } |x + y| = |x - (-y)|
\rightarrow \text{distance between } x, -y.
\]

29. (2 points) Solve for \(x\) if \(|3x - 2| < 4\)

\[-4 < 3x - 2 < 4\]
\[-4 < 3x - 2\]
\[-\frac{2}{3} < x\]

30. (1 point) What is the determinant of the matrix below?

\[
\begin{pmatrix}
2 & -3 \\
1 & -5
\end{pmatrix}
\]

\[
\det \begin{pmatrix}
2 & -3 \\
1 & -5
\end{pmatrix} = 2(-5) - (-3)(1) = -10 + 3 = -7
\]

31. (2 points) Find the product

\[
\begin{pmatrix}
1 & 0 \\
3 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 2 \\
0 & 1
\end{pmatrix} = \begin{pmatrix}
1 & 2 \\
3 & 7
\end{pmatrix}
\]

32. (2 points) What is the inverse of the matrix below?

\[
A = \begin{pmatrix}
1 & 4 \\
2 & 3
\end{pmatrix}
\]

\[
A^{-1} = \frac{1}{\det(1, 4, 2, 3)} \begin{pmatrix}
3 & -4 \\
-2 & 1
\end{pmatrix}
\]

\[
= \frac{1}{1(3) - 4(2)} \begin{pmatrix}
3 & -4 \\
-2 & 1
\end{pmatrix}
\]

Points earned: _______ out of a possible 8 points
33. (1 point) Write the following system of three linear equations in three variables as a matrix equation

\[
2x - y + z = 2 \\
y + 2z = 1 \\
-x + y - z = 0
\]

\[
\begin{pmatrix}
2 & -1 & 1 \\
0 & 1 & 2 \\
-1 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= 
\begin{pmatrix}
2 \\
1 \\
0
\end{pmatrix}
\]

34. (2 points) Solve for \(x, y,\) and \(z\) if

\[
\begin{pmatrix}
-1 & 2 & -1 \\
-2 & 2 & -1 \\
3 & -1 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= 
\begin{pmatrix}
-2 \\
2 \\
1
\end{pmatrix}
\]

\[
\begin{pmatrix}
-1 & 2 & -1 \\
-2 & 2 & -1 \\
3 & -1 & 1
\end{pmatrix}^{-1}
= 
\begin{pmatrix}
1 & -1 & 0 \\
1 & 2 & 1 \\
-4 & 5 & 2
\end{pmatrix}
\]

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= 
\begin{pmatrix}
1 & -1 & 0 \\
-1 & 2 & 1 \\
-4 & 5 & 2
\end{pmatrix}^{-1}
= 
\begin{pmatrix}
-2 + (-2) + 0 = -4 \\
2 + 4 + 1 = 7 \\
8 + 10 + 2 = 20
\end{pmatrix}
= 
\begin{pmatrix}
-4 \\
7 \\
20
\end{pmatrix}
\]
For #35–#40, mark and label x- and y-intercepts (if there are any).

35. (2 points) \(3\)

36. (2 points) \(x\)

37. (2 points) \(x^2\)

38. (2 points) \(x^3\)

39. (2 points) \(\sqrt{x}\)

40. (2 points) \(\sqrt{x}\)
For #41–#46, mark and label $x$- and $y$-intercepts (if there are any).

41. (2 points) $\frac{1}{x}

42. (2 points) $\frac{1}{x^2}

43. (2 points) $e^x

44. (2 points) $\log_e(x)$

45. (2 points) $-2e^{-x}

46. (2 points) $-3\sqrt{x} + 2
47. (2 points) \( f: (-2, 0] \to \mathbb{R} \)
where \( f(x) = x^2 \)

48. (2 points) \( g: \{-4, -2, 2\} \to \mathbb{R} \)
where \( g(x) = \frac{1}{2}x - 1 \)

49. (2 points) \( 4(x + 2)^2 + 1 \)

\[ g(-4) = \frac{1}{2} (-4) - 1 = -2 - 1 = -3 \]
\[ g(-2) = \frac{1}{2} (-2) - 1 = -1 - 1 = -2 \]
\[ g(2) = \frac{1}{2} (2) - 1 = 1 - 1 = 0 \]
50. (2 points) Graph \( p(x) \) and label all \( x \)-intercepts:

\[ p(x) = -2(x + 1)(x + 1)(x - 2)(x^2 + 1) \]

\( x = 0 \)
\[ \rightarrow p(0) = -2(1)(1)(-2)(1) \]
\[ \rightarrow p(0) > 0 \]

51. (2 points) Graph \( r(x) \), and label all \( x \)-intercepts and all vertical asymptotes:

\[ r(x) = \frac{-3(x - 1)(x - 1)}{4(x + 2)(x^2 + 1)} \]

\( x = -2 \)

Points earned: _____ out of a possible 4 points
52. (2 points) $h(x) = \begin{cases} e^x & \text{if } x \neq 1 \\ -3 & \text{if } x = 1 \end{cases}$

53. (2 points) $m(x) = \begin{cases} 1 & \text{if } x \in (-\infty, 1) \\ 3 & \text{if } x = 1 \\ x^2 & \text{if } x \in (1, 2] \end{cases}$