Practice problems for third exam
April 11, 2015

For #1–6, are the statements true or false?

1. $a^x a^y = a^{x-y}$
2. $\frac{a^x}{a^y} = a^{x-y}$
3. $\log_a(zw) = \log_a(z) + \log_a(w)$
4. $\log_a(z^w) = \log_a(z)w$
5. $\log_a(\frac{z}{w}) = \frac{\log_a(z)}{\log_a(w)}$
6. $(a^x)^y = a^{x+y}$

7. Write $5^{0.28}5^{-0.005}5^{-26}$ as a rational number in standard form.

8. Write $(4^{\frac{2}{5}})^{\frac{15}{7}}$ as a rational number in standard form.

9. Write $1,000,000^{\frac{7}{2}}$ as a rational number in standard form.

10. Write $(\frac{9}{4})^{-\frac{3}{2}}$ as a rational number in standard form.
11. Write \( \log_{10}(10,000) \) as a rational number in standard form.

12. Write \( \log_5\left(\frac{1}{\sqrt{125}}\right) \) as a rational number in standard form.

13. What is the greatest integer that is less than \( \log_4(50) \)?

14. Solve for \( x \) if \( e^x = 5 \)

15. Solve for \( x \) if \( \log_4(x) = -3 \). (Write your answer as a rational number in standard form.)

16. Solve for \( x \) if \( 4e^{x-3} = 8 \).
17. Solve for $x$ if $\log_e(x + 2) - 7 = 9$

18. Solve for $x$ if $e^{3x+1} = \frac{4}{e^x}$

19. Solve for $x$ if $\log_e(x^2 + x) - 2 = \log_e(x)$
20. Graph \( p(x) = -3(x + 1)(x - 2)(x - 2)(x^2 + 7) \)

21. Graph \( r(x) = \frac{2(x+2)(x^2+3)}{5(x-2)(x-2)} \)

22. Graph \( e^x \) and label its \( y \)-intercept.
23. Graph $\log_e(x)$ and label its $x$-intercept.

24. Graph $e^x - 1$ and label its $x$- and $y$-intercepts (if there are any).

25. Graph $\log_e(x + 2)$ and label its $x$- and $y$-intercepts (if there are any).
26. Graph \( f : (-1,2] \rightarrow \mathbb{R} \) where \( f(x) = -2x \).

27. Graph \( g : [-2,1) \rightarrow \mathbb{R} \) where \( g(x) = -x^2 \).

28. Graph \( g : \mathbb{R} \rightarrow \mathbb{R} \) where \( g(x) = \begin{cases} -x & \text{if } x \in (-\infty, 0) \\ 2 & \text{if } x = 0 \\ x & \text{if } x \in (0, \infty) \end{cases} \).
29. Graph $g : \mathbb{R} \to \mathbb{R}$ where $g(x) =$ \begin{align*}
  x^2 & \quad \text{if } x \in (-\infty, 0) \\
  2 - x & \quad \text{if } x \in [0, 2) \\
  2 & \quad \text{if } x \in [2, \infty) 
\end{align*}

30. Solve for $x$ if $|4x - 9| < 5$

31. Solve for $x$ if $|-2x + 6| < 4$
32. Consider the following plot depicting a vertical line (in blue):

(a) Does this plot depict the graph of a function?
(b) Does this plot depict the solutions of a linear equation in two variables?

33. What is the slope of the line $45x - 5y = 3$?

34. What is the slope of the line $-18x + 6y = 4$?

35. Is there a unique solution to the following system?

\[
\begin{align*}
45x - 5y &= 3 \\
-18x + 6y &= 4
\end{align*}
\]
36. Find the solution to the system of two linear equations in two variables. (Hint: The system has a unique solution.)

\[
5x - 3y = 4 \\
-7x + y = -20
\]

37. Consider the following system of linear equations in three variables:

\[
-x + 7y - 8z = 10 \\
3y - 5z = 5 \\
x - 4y + 3z = -5
\]

Note: \( x = \frac{16}{3}, y = \frac{10}{3}, z = 1 \) is a solution of the above system.

(a) Is \( x = -2, y = 0, z = -1 \) a solution to the above system?

(b) Does the above system have a unique solution?

(c) Does the above system have infinitely many solutions?