ORIENTED INTERSECTION MULTIPLICITIES

V. SRINIVAS

The "classical" algebraic theory of intersection multiplicites, as developed by Serre and others, is related to the intersection product of rational equivalence classes of algebraic cycles, which gives the multiplication in the Chow ring of algebraic cycles modulo rational equivalence.

One definitive algebraic result proved using the Chow ring is Murthy's Theorem, that over a d-dimensional affine algebra over an algebraically closed field, a projective module of rank d has a trivial direct summand if and only if its d-th Chern class vanishes. This is trivially false over the field of real numbers, since any even dimensional sphere has no vector fields which are nowhere vanishing.

In topology, one has a definition of the Euler class of a real vector bundle, whose vanishing gives a necessary and sufficient condition for the bundle to have a nowhere vanishing continuous section. This inspired a definition of Euler class groups, due to Bhatwadekar and Sridharan, which attempted to define an analogous invariant for rank d projective modules over d-dimensional affine varieties over a field. While this theory is well developed for affine algebras over the real numbers, it seems basically limited to that situation.

There has been recent work by Jean Fasel to develop an oriented intersection theory, based on a brief outline by Jean Barge and Fabian Morel. It is expected that this will reduce to the work of Bhatwadekar-Sridharan in the appropriate situations, while giving a more general theory, with various good formal properties like localization exact sequences, intersections with support, etc.

One definition of the d-th Chow group is as the d-th homology of the d-th Gersten complex in Milnor K-theory, whose terms are direct sums of Milnor K-groups of function fields of subvarieties. The d-th oriented Chow group is similarly defined to be the d-th homology of a complex, the d-th Gersten-Witt complex, built up out of the Milnor K-groups and Witt groups of quadratic forms of function fields of subvarieties. The Witt group aspect of this complex depends on the recent development by Paul Balmer of triangular Witt groups, associated to a triangulated category equipped with a duality, and the machinery for manipulating these, due to Balmer, Gille and others.

Given that one can construct such an oriented Chow ring, it is natural to ask whether there is a related theory of intersection multiplicites. In this talk, I will give an outline of the above oriented intersection theory, and discuss some joint ongoing work with Fasel on such an oriented intersection multiplicity.

School of Mathematics, Tata Institite of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai-400005, India

E-mail address: srinivas@math.tifr.res.in