NUMERICAL EQUIVALENCE AND THE VANISHING CONJECTURE

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In 1985, P. Roberts solved the vanishing conjecture affirmatively for complete intersections. He proved that the limit multiplicities (Dutta multiplicity) vanish under a condition. For complete intersections, the limit multiplicity coincides with the ordinary intersection multiplicity. It is very interesting to study when the limit multiplicity coincides with the ordinary intersection multiplicity. A few years ago, I defined a notion of numerical equivalence on the Chow group and the Grothendieck group of finitely generated modules over a Noetherian local ring. If we divide the Chow group and the Grothendieck group of a Noetherian local ring, we obtain free abelian groups of the same rank. So, we can define an invariant of Noetherian local rings as the Picard number of a smooth projective variety. Differnce between the limit multiplicity and the ordinary intersection multiplicity comes from cycles (of dimension strictly less than the dimension of the given local ring) which are not numerically equivalent to zero. Such a cycle was first discovered by Dutta, Hochster and MacLaughlin. Miller and Singh gave another example. Levin, Roberts and Srinivas explained why such cycles exist using a theory of algebraic cycles or algebraic K-theory. We introduce a notion of numerical equivalence, and give relations to the vanishing conjecture and the standard conjectures.