Ask yourself:

- Can I tell what is the question being answered?
- Can I tell what is being assumed and what is being shown?
- Did the author skip any steps?
- Is it easy to see why each step followed from the last?
- While reading this solution, can I see where the author is going with their work?

Pro tips:

- Pretend you’re teaching someone who’s never seen this problem how to solve it.
- Look at the textbook for examples of good mathematical writing.
1. We wish to find the area of the largest rectangle that can be inscribed in a semicircle of radius $r$.

Let’s consider the semi-circle oriented in the plane as below:

which has equations $x^2+y^2=r^2$, $y \geq 0$.

Let $(x,y)$ be the corner of an inscribed rectangle, then the area of that rectangle is $A = 2xy$.

This is the function we want to maximize, but we need to use the constraint, $x^2+y^2=r^2$, to eliminate a variable:

$x^2+y^2=r^2$
$x^2=r^2-y^2$

Now,

$A = 2xy = 2(\sqrt{r^2-y^2})y$

To find the max we need critical points:

$A = 2y\sqrt{r^2-y^2}$

$\frac{dA}{dy} = 2\sqrt{r^2-y^2} + 2y(\frac{1}{2})(r^2-y^2)^{-\frac{1}{2}}(-2y)$

$= \frac{2(r^2-2y^2)}{\sqrt{r^2-y^2}}$

$2\frac{r^2-2y^2}{\sqrt{r^2-y^2}} = 0 \Rightarrow r^2-2y^2 = 0$

$r^2 = 2y^2$

$y = \pm \frac{r}{\sqrt{2}}$

We’re interested in $y > 0$, so max area is when $y = \frac{r}{\sqrt{2}}$, and is

$A = 2\sqrt{r^2-(\frac{r}{\sqrt{2}})^2}(\frac{r}{\sqrt{2}})$

$= 2\sqrt{\frac{r^2}{2}}(\frac{r}{\sqrt{2}}) = \sqrt{r^2}$.