

Worksheet #2 SOLUTIONS
Math 1100-005

1. Find $\frac{dy}{dx}$ for $3xy^2 + x^2 + y^2 = 2$

$$\begin{aligned}3y^2 + 6xy\frac{dy}{dx} + 2x + 2y\frac{dy}{dx} &= 0 \\6xy\frac{dy}{dx} + 2y\frac{dy}{dx} &= -3y^2 - 2x \\ \frac{dy}{dx}[6xy + 2y] &= -3y^2 - 2x \\ \frac{dy}{dx} &= \frac{-3y^2 - 2x}{6xy + 2y}\end{aligned}$$

2. Find the slope of the tangent line of $2\sqrt[3]{x} + 3\sqrt{y} = 10$ at $(8, 4)$.

$$\begin{aligned}\frac{2}{3}x^{-2/3} + \frac{3}{2}y^{-1/2}\frac{dy}{dx} &= 0 \\ \frac{3}{2}y^{-1/2}\frac{dy}{dx} &= -\frac{2}{3}x^{-2/3} \\ \frac{dy}{dx} &= \frac{-\frac{2}{3}x^{-2/3}}{\frac{3}{2}y^{-1/2}} \\ \frac{-4y^{1/2}}{9x^{2/3}} & \\ \frac{dy}{dx}(8, 4) &= \frac{-4\sqrt{4}}{9\sqrt[3]{8^2}} \\ &= \frac{-2}{9}\end{aligned}$$

3. For $f(x) = (x - 1)^2(x - 3)$,

- (a) determine the intervals for which the function is increasing and decreasing
Find derivative:

$$\begin{aligned}f'(x) &= 2(x - 1)(x - 3) + (x - 1)^2 \\ &= (x - 1)[2(x - 3) + (x - 1)] \\ &= (x - 1)(2x - 6 + x - 1) \\ &= (x - 1)(3x - 7)\end{aligned}$$

Find critical values:

$$\begin{aligned}f'(x) &= (x - 1)(3x - 7) = 0 \\ x &= 1 \\ \text{or} \\ x &= \frac{7}{3}\end{aligned}$$

Intervals	$(-\infty, 1)$	$(1, \frac{7}{3})$	$(\frac{7}{3}, \infty)$
Test Value	$x = 0$	$x = 2$	$x = 3$
Sign of $f'(x)$	+	-	+
Inc/Dec?	INC	DEC	INC

(b) determine the relative extrema

From the table above: relative max at $x = 1$ and relative min at $x = \frac{7}{3}$

4. Find ALL extrema of $h(x) = x^4 - 2x^3$ on $[-1, 2]$.

Finding **ALL** extrema means finding BOTH *relative* AND *absolute* extrema.

For RELATIVE EXTREMA:

- find derivative:

$$\begin{aligned} h'(x) &= 4x^3 - 6x^2 \\ &= 2x^2(2x - 3) \end{aligned}$$

- find critical values:

$$\begin{aligned} h'(x) &= 2x^2(2x - 3) = 0 \\ x &= 0 \\ \text{or} \\ x &= \frac{3}{2} \end{aligned}$$

- find increasing/decreasing intervals:

Intervals	$[-1, 0)$	$(0, \frac{3}{2})$	$(\frac{3}{2}, 2]$
Test Value	$x = -\frac{1}{2}$	$x = 1$	$x = \frac{7}{4}$
Sign of $f'(x)$	-	-	+
Inc/Dec?	DEC	DEC	INC

Then $h(x)$ has a RELATIVE MIN at $x = \frac{3}{2}$

NOW, to find the ABSOLUTE MIN/MAX: test $h(x)$ at $x = -1$ and $x = 2$ (the endpoints of the interval) AND $x = 0$ and $x = \frac{3}{2}$ (the critical values).

$$\begin{aligned} h(-1) &= 3 \\ h(2) &= 0 \\ h(0) &= 0 \\ h\left(\frac{3}{2}\right) &= -1.6875 \end{aligned}$$

Then $h(x)$ has an ABSOLUTE MIN at $x = \frac{3}{2}$ and an ABSOLUTE MAX at $x = -1$.

Relative Max	none
Relative Min	$x = \frac{3}{2}$
Absolute Max	$x = -1$
Absolute Min	$x = \frac{3}{2}$

5. For $g(x) = x^5 - 10x^2$, determine the intervals for which the function is concave up & down.

- find the first and second derivatives:

$$\begin{aligned}g'(x) &= 5x^4 - 20x \\g''(x) &= 20x^3 - 20 \\ &= 20(x^3 - 1)\end{aligned}$$

- find the inflection points:

$$\begin{aligned}g''(x) &= 20(x^3 - 1) = 0 \\x &= 1\end{aligned}$$

- find intervals of concavity:

Intervals	$(-\infty, 1)$	$(1, \infty)$
Test Value	$x = 0$	$x = 2$
Sign of $g''(x)$	-	+
Concave up/down?	down	up

So, $g(x)$ is concave up on $(1, \infty)$ and concave down on $(-\infty, 1)$.