Exam 2 Review
Math 1100-005

Topics to Study

Implicit Differentiation

- Know how to take derivatives implicitly & evaluate those derivatives at a point

First Derivative Test & Extrema

- Know how to find critical points.
- Know how to find increasing/decreasing intervals of a function.
- Know how to find relative AND absolute extrema.

Second Derivative Test

- Know how to find inflection points.
- Know how to find intervals of concavity.
- Know how to use the 2nd derivative test to find relative extrema.

Optimization

- Know how to maximize/minimize certain quantities.
- Know how to solve optimization problems.
- Know how to find elasticity AND regions that are elastic/inelastic.

How to Study

- REDO and make sure to understand ALL HOMEWORK problems(from homeworks 4 and 5)
- DO the optional problems given on the homeworks
- REDO all the examples given in class
- REDO worksheet #2
- DO the following review

Equations GIVEN

$$\begin{aligned}
\text{Perimeter} &= \text{add up lengths of all sides} \\
\text{Area} &= \text{length*width} \\
\text{Volume} &= \text{length*width*height} \\
\text{Surface Area of Rectangle} &= \text{add up areas of all sides} \\
\text{Distance between}(x_1, y_1)\text{and}(x_2, y_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
\text{Area of Circle} &= \pi r^2 \\
\text{Circumference of Circle} &= 2\pi r \\
\text{Volume of Sphere} &= \frac{4}{3}\pi r^3 \\
\text{Surface Area of Sphere} &= 4\pi r^2 \\
\text{Volume of Cylinder} &= \pi r^2 h \\
\text{Surface Area of Cylinder} &= 2\pi r h + 2\pi r^2
\end{aligned}$$

Equations TO KNOW!

$$\begin{aligned}
\text{Profit} &= \text{Total Revenue - Total Cost} \\
\text{Revenue} &= \text{price*quantity} = p(x)x \\
\text{Cost} &= \text{Fixed Costs + Variable Costs} \\
\text{Average Cost} &= \frac{\text{Cost}}{\text{quantity}} \\
\eta &= \frac{\text{change in demand}}{\text{change in price}} = \frac{p/x}{dp/dx} \qquad \text{(Price Elasticity of Demand)}
\end{aligned}$$

Review of the 1st and 2nd Derivative Tests

CRITICAL VALUES: where f'(x)=0 AND f'(x)=undefined

INFLECTION POINTS: where f"(x)=0 AND f"(x)=undefined

What You're Trying to Find	**What You Need**	**Which Derivative to Use**		
Increasing/Decreasing Intervals	Critical Values	1st Deriv: f'(x)	f'(x)>0 (INC)	f'(x)<0 (DEC)
Relative Extrema (using 1st Deriv)	Critical Values	1st Deriv: f'(x)	MAX if INC then DEC	MIN if DEC then INC
Relative Extrema (using 2nd Deriv)	Critical Values	2nd Deriv: f"(x)	f"(c)>0 (MIN)	f"(c)<0 (MAX)
Absolute Extrema	Critical Values End Points [a,b]	function: f(x)	MAX: largest value of f(c),f(a),f(b)	MIN: smallest value of f(c),f(a),f(b)
Concavity	Inflection Points	2nd Deriv: f"(x)	f"(x)>0 (Concave Up)	f"(x)<0 (Concave Down)

Review of Solving Optimization Problems

STEPS:

1. Sketch (if possible)
2. Write equation for the information that is given to you.
3. Write the equation for the value that you're trying to maximize/minimize.
4. Write the equation in step 3 as a function of one variable. This means solving your equation in step 2 for one variable and substituting it into the equation from step 3.
5. Take the derivative of your equation in step 4. Set the derivative equal to zero and solve.
6. Check that the value you got in step 5 is a min or max (whichever the problem was asking for). To do this, you will need to use the 1st OR 2nd derivative test for relative extrema.
7. Finally, solve for any other variables. (Make sure you actually answered the question!!!)

Review Problems - DON'T ONLY study these!!

1. Find $\frac{dy}{dx}$ for

   (a) $(x+y)^3 = x^3 + y^3$

   (b) $y^2 = \frac{x^3}{4-x}$ at $(2,2)$

2. Find the critical values AND inflection points for:

   (a) $f(x) = (x+2)^{2/3}$

   (b) $f(x) = \sqrt{x^2-1}$

3. For $f(x) = 6x^3 - 15x^2 + 12x$, find:

   (a) critical values

   (b) increasing and decreasing intervals

   (c) inflection points

   (d) concavity intervals

   (e) all extrema

4. For $f(x) = x^3 - 3x^2$ on the interval $[-1, 3]$, find:

   (a) critical values

   (b) increasing and decreasing intervals

   (c) inflection points

   (d) concavity intervals

   (e) all extrema

5. For $f(x) = x^3 - 3x$,

   (a) find the relative extrema using the 1st derivative test.

   (b) verify what you found in part (a) with the 2nd derivative test.

6. Find 2 positive numbers such that the sum of the first and twice the second is 100 and the product is a maximum.

7. The combined perimeter of a circle and a square is 16. Find the dimensions of the circle and square that produce a minimum total area.

8. For $0.3x^2 + 6x + 600$, find the average cost for producing 25 units.