Exam 1 Review Answers (NOT solutions)
Math 1100-005

Review Problems

1. Find the limit (if it exists):

   (a) $\lim\limits_{x \to -3} \frac{x^2-x-12}{x+3} = -7$

   (b) $\lim\limits_{t \to 9} \frac{9-t}{3-\sqrt{t}} = 6$

2. For the following equation, find the limits (if they exist):

$$h(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$$

   (a) $\lim\limits_{x \to 0^+} h(x) = 0$

   (b) $\lim\limits_{x \to 0} h(x) = 0$

   (c) $\lim\limits_{x \to 1} h(x) = 1$

   (d) $\lim\limits_{x \to 2^+} h(x) = 6$

   (e) $\lim\limits_{x \to 2^-} h(x) = 4$

   (f) $\lim\limits_{x \to 2} h(x) =$ d.n.e

3. Describe the **intervals** on which the function is continuous:

   (a)

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq 2 \\ x^2 - 2x & \text{if } x > 2 \end{cases}$$

   Continuous Intervals: $(-\infty, 2)$ and $(2, \infty)$

   (b)

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

   Continuous Intervals: $(-\infty, 0)$ and $(0, \infty)$

4. Write the equation to find the slope of a tangent line.
   $m = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

5. Explain what a derivative means (include why we take derivatives).
   A derivative is the slope/rate of change of a graph/function. We take derivatives to see the rate at which our function/graph is changing at a certain point. Derivatives are useful in many applications including velocity, acceleration, and marginals.

6. Find the derivative using the definition of a derivative(**simplify** as much as possible):

   (a) $f(x) = \frac{1}{x}$
   Use $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ to find that $f'(x) = \frac{-1}{x^2}$

7. Find the derivative:

   (a) $f(x) = (5x^2 - 2)^{25}$
   $f'(x) = 250x(5x^2 - 2)^{24}$

   (b) $y = \frac{x^2}{\sqrt{6x-x^2}}$
   $y' = \frac{2x(6x-x^2)^{1/2} - x^2(\frac{1}{2})(6x-x^2)^{-1/2}(6-2x)}{6x-x^2}$

   (c) $f(x) = x^4 - 2x^2 + 2$
   $f'(x) = 4x^3 - 4x$

   (d) $y(x) = \sqrt{x^3 - 2x^2 - 1}$
   $y'(x) = \frac{1}{2}(x^3 - 2x^2 - 1)^{-1/2}(3x^2 - 4x)$

   (e) $f(x) = (x^2 - 2x)^2(x+3)^5$
   $f'(x) = 2(x^2 - 2x)(2x - 2)(x+3)^5 + 5(x^2 - 2x)^2(x+3)^4$

8. For $f(x) = x^{10} - 6x^3 + \frac{1}{x}$, find the indicated derivatives:

   (a) $f'(x) = 10x^9 - 18x^2 - \frac{1}{x^2}$

   (b) $f^{(4)}(x) = 5040x^6 + \frac{24}{x^5}$

9. Suppose that the height of a projectile fired vertically upward from a height of 64 feet with an initial velocity of 48 feet per second is given by $h(t) = -16t^2 + 48t + 64$

   (a) Find the height of the object at 3 seconds.
   $h(3) = 64ft$

   (b) What is the average velocity of the projectile for the time interval from t=0 to t=1.5?
   Avg Velocity $= 24ft/s$

   (c) At what time does the object reach is maximum height?
   $t = 1.5s$

   (d) How fast is the object traveling when it hits the ground?
   $-80ft/s$

   (e) What is the projectile's acceleration?
   $-32ft/s^2$

10. Describe the difference between average, marginal, and instantaneous.
    Average is the "average" between 2 points, ie. $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$. Both marginal and instantaneous use the same mathematical idea, the derivative. They are both finding the rate of change at some specific point/value rather than between 2 points/values. Marginals are associated with business applications, ie. Profit, Revenue and Cost, while instantaneous refers more to velocity and acceleration.

11. Write general equations for profit, cost and revenue.
    Profit=Total Revenue - Total Cost
    Revenue = price*quantity = p(x)*x
    Cost = Fixed costs + Variable costs