Exam 3 Review
Math 1100-005

Topics to Study

Asymptotes & Sketching

- Know how to find both vertical and horizontal asymptotes
- Know how to sketch a graph by finding
  - intercepts
  - increasing/decreasing intervals
  - extrema
  - concavity
  - asymptotes
- Know how to sketch a graph if given information about it (ie. information about 1st and 2nd derivatives, asymptotes, etc.)

Derivatives

- Know how to find derivatives of algebraic, exponential & log functions

Integrals

- Know how to compute indefinite integrals
- Know how to find business functions if given their marginals (ie. cost, revenue, profit, price)

How to Study

- REDO and make sure to understand ALL HOMEWORK problems(from homeworks 6 and 7)
- DO the optional problems given on the homeworks
- REDO all the examples given in class
- REDO worksheet #3
- DO the following review

Equations TO KNOW!

$$\begin{aligned} \text{Profit} &= \text{Total Revenue - Total Cost} \\ \text{Revenue} &= \text{price*quantity} = p(x)x \\ \text{Cost} &= \text{Fixed Costs + Variable Costs} \\ \text{Average Cost} &= \frac{\text{Cost}}{\text{quantity}} \end{aligned}$$

VERTICAL Asymptotes

- Vertical asymptotes occur where the function is UNDEFINED:

- EXAMPLES of where functions are undefined at values of x include when:
  - the denominator equals 0
    (ie. for $f(x) = \frac{1}{x}$, $f(x)$ is undefined at $x = 0$)
  - the interior of a log function is 0 or negative
    (ie. for $f(x) = ln(x)$, $f(x)$ is undefined at $x \leq 0$)
  - the interior of a square root is negative
    (ie. for $f(x) = \sqrt{x}$, $f(x)$ is undefined at $x < 0$)

HORIZONTAL Asymptotes

- To find horizontal asymptotes:
  EVALUATE THE LIMITS: $\lim_{x\to\infty}$ AND $\lim_{x\to-\infty}$

- Some rules & tricks for finding horizontal asymptotes:
  - For algebraic functions (functions without logs or exponentials):
    - if degree of numerator > degree of denominator, then NO horizontal asymptote
    - if degree of numerator = degree of denominator, then HA is $y = \frac{a}{b}$
      where a=leading coefficient of numerator and b=leading coefficient of denominator
    - if degree of numerator < degree of denominator, then HA is $y = 0$.
    - remember that $\frac{1}{big} = small$ (ie. $\lim_{x\to\infty} \frac{1}{x} = 0$)
  - For exponential functions
    - REMEMBER that $e^{\infty} \approx \infty$
    - REMEMBER that $e^{-\infty} \approx 0$
  - For log functions
    - REMEMBER that $\ln(\infty) \approx \infty$
    - REMEMBER that $\ln(-\infty)$ is undefined!
  - Remember that you can sometimes use L'Hopitals Rule: $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$

Review Problems - DON'T ONLY study these!!

1. For the following function, find both vertical and horizontal asymptotes:

   (a) $f(x) = \frac{3x^2+100}{4x^2-100}$

2. Find the derivative of the following functions.

   (a) $f(x) = 1 - e^{x+\frac{1}{x}} + x^2 e^{2x}$

   (b) $f(x) = \ln\left(\frac{x^2-1}{x^3-3x-1}\right)$

3. Evaluate the following integrals.

(a) $\int x + \frac{1}{x} dx$

(b) $\int e^{2x+5} - 3x^3 - 1 dx$

(c) $\int \frac{3x^2-1}{4x^3-4x} dx$

(d) $\int (x+1)\sqrt{x^2+2x-1} dx$

4. The marginal revenue for the sale of a product can be modeled $\frac{dR}{dt} = 50 - 0.02x + \frac{100}{x+1}$ where x is the quantity in demand. If $R = 0$ when $x = 0$, find the revenue function.

5. For $f(x) = \frac{x-3}{x}$,

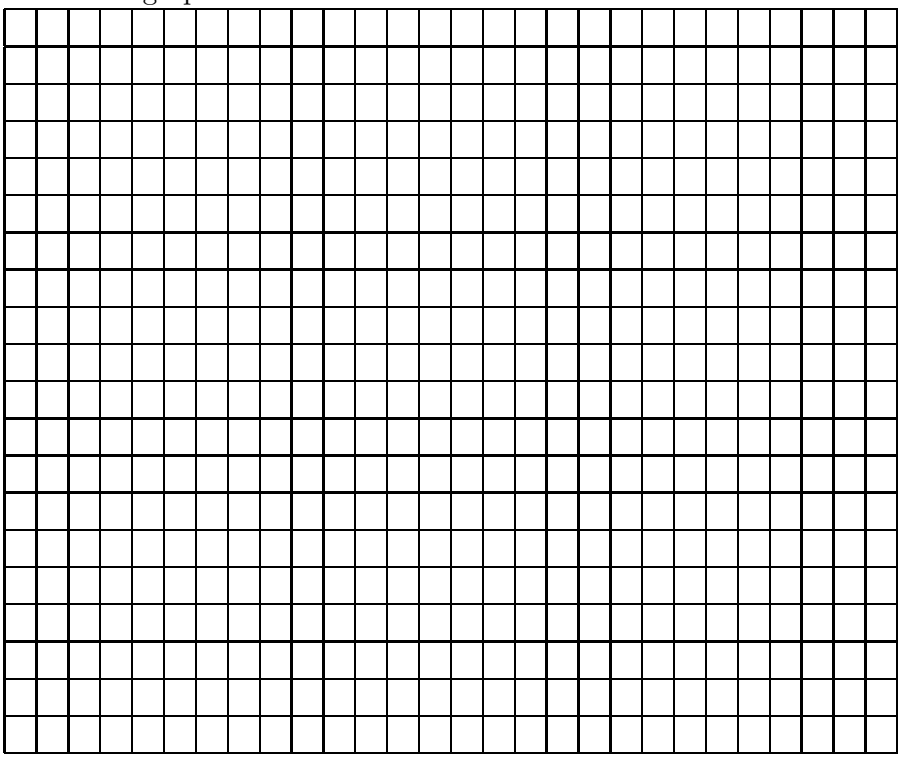
   (a) find the x and y intercepts.

   (b) find the increasing/decreasing intervals and relative extrema.

   (c) find the concavity intervals.

(d) find the vertical and horizontal asymptotes.

(e) sketch the graph



6. Tell what each of the following mean:

- f'(x)>0

- f"(x)<0 for x>1

- $\lim_{x \to -\infty} f(x) = 2$