

Exam 3 Review SOLUTIONS
Math 1100-005

1. For the following function, find both vertical and horizontal asymptotes:

(a) $f(x) = \frac{3x^2+100}{4x^2-100}$

Vertical Asymptotes (where $f(x)$ is undefined \rightarrow set denominator equal to zero)

$$\begin{aligned} 0 &= 4x^2 - 100 \\ 100 &= 4x^2 \\ 25 &= x^2 \\ x = 5 \text{ AND } x = -5 &\text{ are vertical asymptotes} \end{aligned}$$

Horizontal Asymptotes:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x^2 + 100}{4x^2 - 100} &= \frac{3}{4} \\ \lim_{x \rightarrow -\infty} \frac{3x^2 + 100}{4x^2 - 100} &= \frac{3}{4} \end{aligned}$$

Then the horizontal asymptote is $y = \frac{3}{4}$

2. Find the derivative of the following functions.

(a) $f(x) = 1 - e^{x+\frac{1}{x}} + x^2 e^{2x}$

$$\begin{aligned} f'(x) &= -e^{x+\frac{1}{x}} \left(1 - \frac{1}{x^2}\right) + 2xe^{2x} + 2x^2 e^{2x} \\ &= -e^{x+\frac{1}{x}} \left(1 - \frac{1}{x^2}\right) + 2x(1+x)e^{2x} \end{aligned}$$

(b) $f(x) = \ln\left(\frac{x^2-1}{x^3-3x-1}\right)$

First simplify $f(x)$

$$\begin{aligned} f(x) &= \ln\left(\frac{x^2-1}{x^3-3x-1}\right) \\ &= \ln(x^2-1) - \ln(x^3-3x-1) \end{aligned}$$

Now, take the derivative

$$f'(x) = \frac{2x}{x^2-1} - \frac{3x^2-3}{x^3-3x-1}$$

3. Evaluate the following integrals.

(a) $\int x + \frac{1}{x} dx$

$$\int x + \frac{1}{x} dx = \frac{x^2}{2} + \ln(x) + C$$

(b) $\int e^{2x+5} - 3x^3 - 1 dx$

Need u-substitution for e^{2x+5} part:

$$\begin{aligned}u &= 2x + 5 \\ \frac{du}{dx} &= 2 \rightarrow \frac{du}{2} = dx\end{aligned}$$

Then, splitting up the integrals and substituting in u and $\frac{du}{2}$

$$\begin{aligned}\int e^{2x+5} - 3x^3 - 1 dx &= \int e^{2x+5} dx - \int 3x^3 dx - \int 1 dx \\ &= \int e^u \frac{du}{2} - \int 3x^3 dx - \int 1 dx \\ &= \frac{e^u}{2} - \frac{3x^4}{4} - x + C\end{aligned}$$

Now, substituting $u = 2x + 5$ back in for u

$$\int e^{2x+5} - 3x^3 - 1 dx = \frac{e^{2x+5}}{2} - \frac{3x^4}{4} - x + C$$

(c) $\int \frac{3x^2-1}{4x^3-4x} dx$

Need u-substitution:

$$\begin{aligned}u &= 4x^3 - 4x \\ \frac{du}{dx} &= 12x^2 - 4 \rightarrow \frac{du}{4} = (3x^2 - 1) dx\end{aligned}$$

Then, substituting in u and $\frac{du}{4}$

$$\begin{aligned}\int \frac{3x-1}{4x^3-4x} dx &= \int \frac{1}{4u} du \\ &= \frac{1}{4} \ln(u) + C\end{aligned}$$

Now, substituting $u = 4x^3 - 4x$ back in for u

$$\int \frac{3x-1}{4x^3-4x} dx = \frac{1}{4} \ln(4x^3 - 4x) + C$$

(d) $\int (x + 1)\sqrt{x^2 + 2x - 1} dx$

Need u-substitution:

$$\begin{aligned}u &= x^2 + 2x - 1 \\ \frac{du}{dx} &= 2x + 2 \rightarrow \frac{du}{2} = (x + 1)dx\end{aligned}$$

Then, substituting in u and $\frac{du}{2}$

$$\begin{aligned}\int (x + 1)\sqrt{x^2 + 2x - 1} dx &= \int \frac{\sqrt{u}}{2} du \\ &= \frac{u^{3/2}}{3} + C\end{aligned}$$

Now, substituting $u = x^2 + 2x - 1$ back in for u

$$\int (x + 1)\sqrt{x^2 + 2x - 1} dx = \frac{(x^2 + 2x - 1)^{3/2}}{3} + C$$

4. The marginal revenue for the sale of a product can be modeled $\frac{dR}{dx} = 50 - 0.02x + \frac{100}{x+1}$ where x is the quantity in demand. If $R = 0$ when $x = 0$, find the revenue function.

To find the revenue function, we need to integrate the marginal:

$$\begin{aligned}R(x) &= \int 50 - 0.02x + \frac{100}{x+1} dx \\ &= 50x - \frac{0.02x^2}{2} + 100 \ln(x+1) + K \\ &= 50x - 0.01x^2 + 100 \ln(x+1) + K\end{aligned}$$

Next step is to solve for K using the fact that $R = 0$ when $x = 0$:

$$\begin{aligned}R(0) &= 50(0) - 0.01(0)^2 + 100 \ln(0+1) + K = 0 \\ K &= 0\end{aligned}$$

Then

$$R(x) = 50x - 0.01x^2 + 100 \ln(x+1)$$

5. For $f(x) = \frac{x-3}{x}$,

(a) find the x and y intercepts.

x-intercepts: (3, 0)

y-intercepts: NONE (x=0 isn't in the domain)

(b) find the increasing/decreasing intervals and relative extrema.

Critical value: $x = 0$

Intervals	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	+	+
Inc/Dec	INC	INC

No relative extrema.

(c) find the concavity intervals.

Inflection point: $x = 0$

Intervals	$(-\infty, 0)$	$(0, \infty)$
Test Value	$x = -1$	$x = 1$
Sign of $f''(x)$	+	-
Concavity	UP	DOWN

(d) find the vertical and horizontal asymptotes.

Vertical Asymptote: $x = 0$

Horizontal asymptote:

$$\lim_{x \rightarrow \infty} \frac{x-3}{x} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x-3}{x} = 1$$

Then the horizontal asymptote is $y = 1$

(e) sketch the graph

SKETCH NOT PROVIDED

6. Tell what each of the following mean:

- $f'(x) > 0$

This means that $f(x)$ is increasing.

- $f''(x) < 0$ for $x > 1$

This means that $f(x)$ is concave down for $(1, \infty)$.

- $\lim_{x \rightarrow -\infty} f(x) = 2$

This means that $f(x)$ has a horizontal asymptote as $x \rightarrow -\infty$ at $y=2$.

This doesn't say anything about the function on the right hand side of the graph.