

MATH 1100-005 EXAM 1 (100 Points)

For full credit:  
SHOW all work & LABEL all graphs!  
BOX all answers!

Name(1pt):\_\_\_\_\_

1. (6pts) Find the limits (if they exist):

(a)  $\lim_{x \rightarrow -2} \frac{x^3 - 2x - 2}{x^2 + 4}$

(b)  $\lim_{t \rightarrow -1} \frac{t+1}{t^3 - t}$

2. (8pts) For the following equation

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ x - 1 & \text{if } x \geq 0 \end{cases}$$

(a) Find  $\lim_{x \rightarrow 0^-} f(x)$ , if it exists.

(b) Find  $\lim_{x \rightarrow 0} f(x)$ , if it exists.

(c) Determine the **interval(s)** of continuity.

3. (6pts) Find the **interval(s)** of continuity:

(a) for the function  $f(x) = \frac{x+1}{x^2-1}$

(b) for the following graph

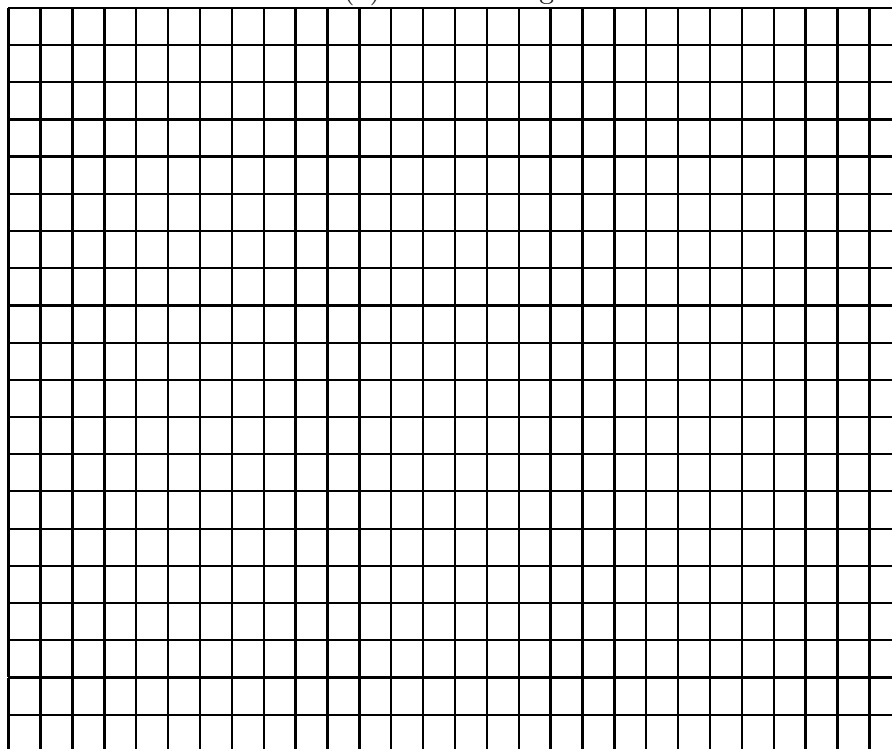
4. (8pts) Use the definition of the derivative to show that the derivative of  $g(x) = 4x^2 - 2x$  is  $g'(x) = 8x - 2$ .

5. (10 pts) For the equation,  $f(x) = \sqrt{x} + 1$ :

(a) Find the equation for the slope of the tangent line.

(b) Find the slope at the point (1, 2).

(c) Sketch both the function  $f(x)$  and the tangent line.



6. (5 pts) Find the derivative of  $f(x) = (5x^5 + 2) \left(x^{2/3} - \frac{1}{x}\right)$  using the product rule. **(DO NOT SIMPLIFY)**.

7. (5 pts) Find the derivative of  $g(x) = \frac{x^4 - 5x^2 - 3}{1 - \sqrt[3]{x}}$  using the quotient rule. **(DO NOT SIMPLIFY)**.

8. (25 pts) Find the following derivatives **(DO NOT SIMPLIFY)**:

(a) For  $f(x) = x^2 + 2x - 1$ , find  $f'(x)$ .

(b) For  $s(x) = \sqrt{x} + \frac{1}{x} + 3$ , find  $s'(x)$ .

(c) For  $h(x) = \left(\frac{4}{3}x^7 - x^3 + 9x\right)^{101}$ , find  $h'(x)$ .

(d) For  $g(y) = \frac{(y^2+3)^2}{y^3-2y-1}$ , find  $g'(y)$ .

(e) For  $f(t) = t^2 + \frac{3}{t} + 5t$ , find  $f'''(t)$ .

9. (10 pts) The equation for motion of a particle is given by  $s(t) = t^3 - 3t$  where  $s$  is in meters and  $t$  is in seconds.

(a) Find the velocity and acceleration functions.

(b) Find the acceleration when the particle's velocity is 0.

10. (16 pts) Given that the demand (price) function for selling glasses of lemonade is given by  $p(x) = 1.75 - 0.0025x$  and the cost function is  $C(x) = 20 + 0.05x$ ,

(a) Write the revenue and profit functions.

(b) Find the marginal cost for producing 300 glasses of lemonade.

(c) Find the marginal profit when 300 glasses of lemonade are sold.

11. **(5pts) EXTRA CREDIT**

Find the derivative of  $f(x) = (x^2 - 1)\left(\frac{1}{x^2} + 2x - 1\right)(x^8 - x^7)^2$ .

12. **(5pts) EXTRA CREDIT**

A population of bacteria is introduced into a culture. The number of bacteria,  $P$ , can be modeled by  $P(t) = 500\left(1 + \frac{4t}{50+t^2}\right)$  where  $t$  is time in hours. Find the rate of change of the population when  $t=2$  hours.