

Math1090 Final Exam
Fall, 2004

Name _____

Instructions:

- q Show all work as partial credit will be given where appropriate.
- q If no work is shown, there may be no credit given.
- q All final answers should be written in the space provided and in simplified form.
- q Each question is worth 10 points.
- q There are 6 parts to this test. For each part, choose two out of three questions to complete. Indicate clearly which two questions you want graded. (Don't leave it to chance!)

Grade

1A	
1B	
1C	
2A	
2B	
2C	
3A	
3B	
3C	
4A	
4B	
4C	
5A	
5B	
5C	
6A	
6B	
6C	

Total

Part 1: Choose two out of three problems to complete.

1A) Find the inverse of the following matrix, if possible. If it's not possible, then explain why.

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

$$A^{-1} = \underline{\hspace{4cm}}$$

1B) For $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$, perform the indicated matrix operations, if possible. If not possible, explain why.

(a) $A + A^T$

$$A + A^T = \underline{\hspace{4cm}}$$

(b) BC

$$BC = \underline{\hspace{4cm}}$$

1C) Use Gauss-Jordan Elimination to solve the following system.

$$2x - 4y + 2z = -4$$

$$4x - 9y + 7z = 2$$

$$-2x + 4y - 3z = 10$$

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$

$$z = \underline{\hspace{2cm}}$$

Part 2: Choose two out of three problems to complete.

2A) Given the arithmetic sequence
-2, 1, 4, 7, 10, ...

(a) Find the 100th term.

100th term = _____

(b) Find the sum of the first 100 terms.

Sum of first 100 terms = _____

2B) How much would have to be invested at the end of each year at 6% interest compounded annually to pay off a debt of \$80,000 in 10 years?

\$ _____

2C) A lottery prize worth \$1,000,000 is awarded in payments of \$10,000 five times a year for 20 years. Suppose the money is worth 20% compounded 5 times per year.

(a) What is the interest rate, i ?

i = _____

(b) What is the number of compoundings, n ?

n = _____

(c) What is the formula used to find the present value of this prize?

(d) What is the present value of this prize?

\$ _____

Part 3: Choose two out of three problems to complete.

3A) For $f(x) = \frac{1}{x}$ and $g(x) = x^2 + 1$

(a) State the domain for both functions.

Domain for $f(x)$ _____

Domain for $g(x)$ _____

(b) Find $g \circ f$ and state the domain of this new function.

$(g \circ f)(x) =$ _____

domain: _____

(c) Find $\frac{f}{g}$.

$\frac{f}{g}(x) =$ _____

3B) Solve the equation.

$$\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}$$

$x =$ _____

3C) Find the equation of the line passing through the points (-1, 1) and (2, 3).

Part 4: Choose two out of three problems to complete.

4A) Graph the linear inequality.

$$-4x < 6y$$

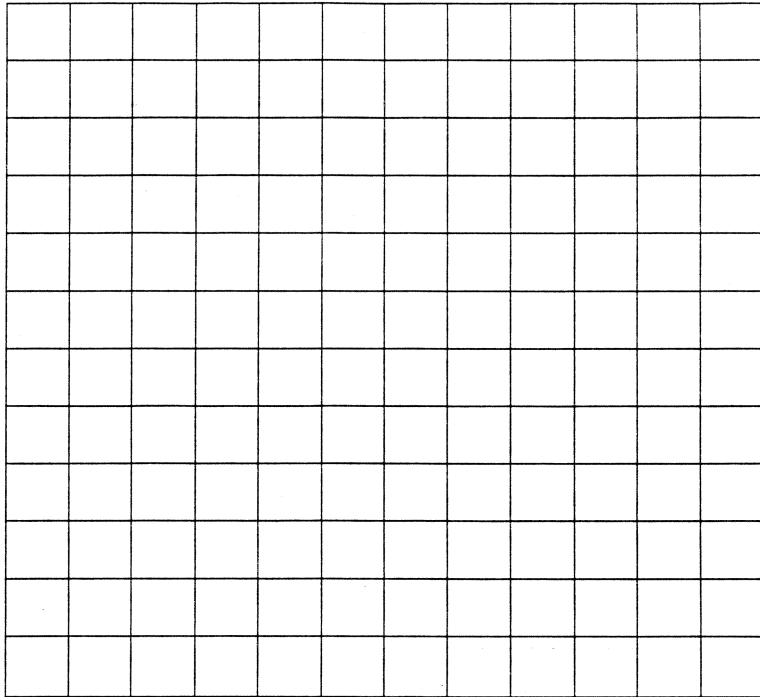
4B) Graph the system of inequalities and shade the solution region. Label all vertices for the solution (shaded) region.

$$3x + 4y \geq 12$$

$$x - y \geq 2$$

$$x \leq 6$$

$$y \geq 0$$



Vertices: _____

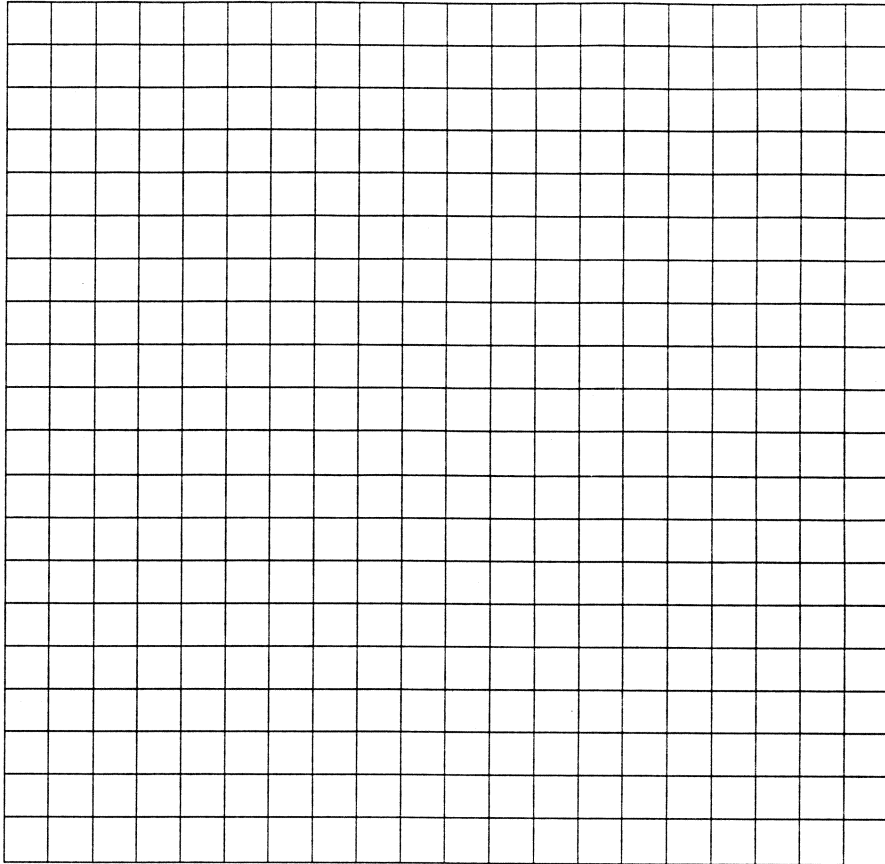
4C) Find the maximum of the objective function $f(x, y) = 2x + y$ subjected to the following constraints.

$$x \geq 0$$

$$y \geq 0$$

$$x + y \leq 10$$

$$2x + 3y \leq 24$$



Maximum value: _____ at point _____

Part 5: Choose two out of three problems to complete.

5A) If the cost of production for a product is given by $C(x)$ and the revenue is given by $R(x) = 30x$,

(a) Find the profit function $P(x)$.

$P(x) =$ _____

(b) Find the break-even point(s).

Break-even point(s): _____

5B) If 100 feet of fence is used to fence in a rectangular yard, then the resulting area is given by

$$A(x) = x(50 - x)$$

where x feet is the width of the rectangle and $(50 - x)$ feet is the length. Determine the width and length that give the maximum area.

Width for max area = _____

Length for max area = _____

5C) Let $f(x) = -(x - 1)^2 + 4$.

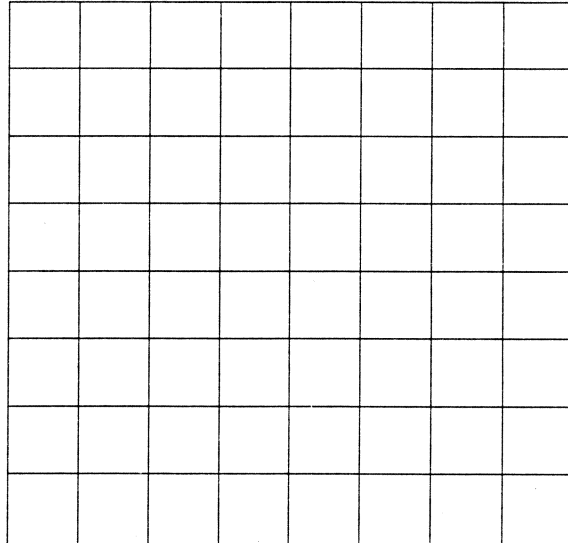
(a) Solve $f(x) = 0$ to find the x-intercepts.

x-intercepts: _____

(b) Find the vertex of the parabola.

Vertex _____

(c) Sketch the graph, showing the vertex and x-intercepts.



Part 6: Choose two out of three problems to complete.

6A) Suppose that the population of Smalltown, USA grows according to the formula

$$P(t) = 3200e^{0.025t}$$

where time t is measured in years.

(a) What is the initial population of the town (at $t = 0$)?

Initial population = _____

(b) How long will it take the population to double?

_____ years

(c) What is the population after 1 year?

6B) Use the properties of logarithms and the fact that

$$\log_{10} 2 \approx 0.3 \quad \log_{10} 5 \approx 0.7 \quad \log_{10} 7 \approx 0.85$$

to find the values below.

(a) $\log_{10} 8$

$\log_{10} 8 \approx$ _____

(b) $\log_{10} 35$

$\log_{10} 35 \approx$ _____

(c) $\log_5 2$

$\log_5 2 \approx$ _____

6C) Rewrite $\log_2 32 = x$ in exponential form and solve for x .

Exponential form: _____

$x =$ _____