MATH 1100-001

REVIEW PROBLEMS FOR EXAM 2

Note:. Exam 2 will cover sections 9.8, 9.9, and 10.1 through 10.5 of the text, and will be on Wednesday, October 28. On the exam you will be allowed one side of a 8.5 by 11 inch sheet of notes. Calculators and other electronic devices are not allowed. The following should not be considered a practice exam (I haven't yet checked it carefully for length and overall difficulty), but they are good review questions, and are the sort of problems that you should be able to do.

1. Suppose that the cost function for producing x units of a particular good is given by $C(x) = \frac{1}{5}x^2 + 4x + 57$ and that the price per unit is $p(x) = \frac{1}{4}(48 - x)$ (the price at which x goods will sell; that is, the demand equation).

a. Find the revenue function.

b. Find the marginal cost and marginal revenue functions.

c. Use the marginal revenue to estimate the revenue derived from the sale of the 31st unit.

d. Find the profit function.

e. At what number of goods sold, will the profit be maximized?

2. Consider the function $f(x) = \frac{1}{4}x^4 - 2x^2 + 2$

a. Find the critical values of f.

b. Determine the relative maximum and minimum points (if any).

c. Determine the intervals on which f is increasing, and decreasing.

d. Determine the intervals on which f is concave up and concave down.

3. Let f be the function

$$f(x) = \frac{1}{8}x + \frac{1}{2x^2}$$

Find the absolute minimum and maximum values of f on the interval $[1, 3]$ and where they occur.

4. Consider the function

$$\frac{2x^2 + 1}{x^2 - 4}$$

a. Find the domain of $f(x)$.

b. Find the vertical and horizontal asymptotes (if any).

c. Using that the derivative is ggiven by

$$f'(x) = \frac{-18x}{(x^2 - 4)^2}$$

find the relative maximum and minimum points.

d. Sketch a graph of $f(x)$.

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Solutions to Review Problems for Exam 2

1. $C(x) = \frac{1}{5}x^2 + 4x + 57$

$p(x) = \frac{1}{4}(48 - x)$

a. $R(x) = x\,p(x) = \boxed{\frac{x}{4}(48-x)}$ or $\boxed{12x - \frac{x^2}{4}}$

b. $MC = \boxed{C'(x) = \frac{2}{5}x + 4}$

$MR = R'(x) = 12 - \frac{2x}{4} = \boxed{12 - \frac{x}{2}}$

c. Revenue from sale of 31st unit is approximately $R'(30)$

or $12 - \frac{30}{2} = 12 - 15 = \boxed{-3}$

d. $P(x) = R(x) - C(x) = 12x - \frac{x^2}{4} - \left(\frac{1}{5}x^2 + 4x + 57\right)$

e. Find critical value:

$P'(x) = R'(x) - C'(x) = 12 - \frac{x}{2} - \left(\frac{2}{5}x + 4\right) = 12 - 4 - \frac{x}{2} - \frac{2}{5}x$

$= 8 - \frac{9}{10}x = 0$

$8 = \frac{9}{10}x \implies \boxed{x = \frac{80}{9}}$

$P''(x) = -\frac{9}{10}$ concave down

so $x = \frac{80}{9}$ gives maximum.

2. $f(x) = \frac{1}{4}x^4 - 2x^2 + 2$

a. $f'(x) = x^3 - 4x$. Critical values: $x^3 - 4x = 0$

$x(x^2 - 4) = 0 \qquad x(x+2)(x-2) = 0$

$\Rightarrow \boxed{x = 0, 2, -2}$

b. First derivative test: $x < -2$: $f'(-3) = -27 + 12 < 0$

$-2 < x < 0$: $f'(-1) = -1 + 4 > 0$

$0 < x < 2$: $f'(1) = 1 - 4 < 0$

$2 < x$: $f'(3) = 27 - 12 > 0$

f': $-$ $\;$ $+$ $\;$ $-$ $\;$ $+$

$-2 \qquad 0 \qquad 2$

$\Rightarrow f(-2) = \frac{(-2)^4}{4} - 2(-2)^2 + 2 = 4 - 8 + 2 = -2$

$f(0) = 2$

$f(2) = -2$

$\Rightarrow (-2, f(-2)) = \boxed{(-2, -2) \text{ relative min. point.}}$

$(0, f(0)) = \boxed{(0, 2) \text{ relative max. point}}$

$(2, f(2)) = \boxed{(2, -2) \text{ relative minimum point.}}$

c. f increasing on $(-2, 0)$ and $(2, \infty)$

f decreasing on $(-\infty, -2)$ and $(0, 2)$.

d. $f''(x) = 12x^2 - 4 = 0$

$$12x^2 = 4$$

$$x^2 = 4/12 = \frac{4}{3}$$

$$x = \pm \frac{2}{\sqrt{3}}.$$

f''	$+$		$-$		$+$
		$-\frac{2}{\sqrt{3}}$		$\frac{2}{\sqrt{3}}$	

$x < -\frac{2}{\sqrt{3}}$ $\quad f''(-2) = 48 - 4 > 0$

$-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$ $\quad f''(0) = -4 < 0$

$\frac{2}{\sqrt{3}} < x$ $\quad f''(2) = 48 - 4 > 0$

concave up on $(-\infty, -\frac{2}{\sqrt{3}})$ and $(\frac{2}{\sqrt{3}}, \infty)$

concave down on $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$.

3. $f(x) = \frac{1}{8}x + \frac{1}{2x^2} = \frac{1}{8}x + \frac{x^{-2}}{2}$

$f'(x) = \frac{1}{8} - \frac{2x^{-3}}{2} = \frac{1}{8} - \frac{1}{x^3}$

Critical values: $\frac{1}{8} - \frac{1}{x^3} = 0 \Rightarrow \frac{1}{8} = \frac{1}{x^3} \Rightarrow x^3 = 8$

$$x = \sqrt[3]{8} = 2.$$

3 values where $f(x)$ might have absolute max/min are

$x = 1, 2, 3$

$$f(1) = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$$

$$f(2) = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$$

$$f(3) = \frac{3}{8} + \frac{1}{18}$$

f has an absolute minimum of $\frac{3}{8}$ at $x=2$

f has an absolute max of $\frac{5}{8}$ at $X=1$.

4. $$f(x) = \frac{2x^2+1}{x^2-4}$$

a. the domain of f is all x except $x^2-4=0$

or $\boxed{x \neq \pm 2}$

b. Vertical Asymptotes: denominator $x^2-4=0$ at $x = \pm 2$. Since numerator non zero,

$\boxed{x=-2, \quad x=2}$ are vertical asymptote

Horizontal Asymptote: $\lim\limits_{x\to\infty} \frac{2x^2+1}{x^2-4} = \lim\limits_{x\to\infty} \frac{2x^2}{x^2} = 2$

$\boxed{y=2}$

c. $f'(x) = \dfrac{-18x}{(x^2-4)^2}$ $f'(x) = 0 \Rightarrow -18x = 0$

$\Rightarrow x = 0.$

critical value is $x = 0$. If $x < 0$ $(x \neq -2)$

then $f'(x) > 0$

If $x > 0$ (but $x \neq 2$) then $f'(x) < 0$.

$\Rightarrow$ relative max point at $(0, f(0)) = \boxed{(0, -\frac{1}{4})}$

No relative min. points.

d.