# Solutions to Review Problems

1. (5 pts each) For $f(x)=\dfrac{x-3}{9-x^2}$, answer the following questions.

(a) $\lim\limits_{x \to 3} f(x) =$ $-1/6$

$$\lim_{x \to 3} \frac{x-3}{9-x^2} = \lim_{x \to 3} \frac{x-3}{(3-x)(3+x)} = \lim_{x \to 3} \frac{-\cancel{(3-x)}}{\cancel{(3-x)}(3+x)} = \frac{-1}{6}$$

(b) $\lim\limits_{x \to -3} f(x) =$ D.N.E.

$$\lim_{x \to -3} \frac{x-3}{9-x^2} = \lim_{x \to -3} \frac{-(3-x)}{(3-x)(3+x)} = \lim_{x \to -3} \frac{-1}{3+x}$$

. does not exist. get $-\frac{1}{0}$

(c) $\lim\limits_{x \to 0} f(x) =$ $-\frac{1}{3}$

$$\lim_{x \to 0} \frac{x-3}{9-x^2} = \frac{0-3}{9-0} = -3/9 = -1/3$$

No zero in denominator

(d) Where is $f(x)$ discontinuous? Only at points where denominator is zero since it is a rational function.

$x = 3, \ x = -3$

2. (5 pts each) Find the following limits.

(a) $\lim_{x \to -\infty} \dfrac{x^2+5x-3}{2x^2+7x} =$ $\frac{1}{2}$

$$\lim_{x \to -\infty} \frac{\frac{1}{x^2}(x^2+5x-3)}{\frac{1}{x^2}(2x^2+7x)} = \lim_{x \to -\infty} \frac{1+\frac{5}{x} \to 0 - \frac{3}{x^2} \to 0}{2+\frac{7}{x} \to 0} = \frac{1}{2}$$

(b) $\lim_{x \to \infty} \dfrac{x^3+x-2}{3x^5+4x^2+1} =$ $0$

$$\lim_{x \to \infty} \frac{\frac{1}{x^5}(x^3+x-2)}{\frac{1}{x^5}(3x^5+4x^2+1)} = \lim_{x \to \infty} \frac{\frac{1}{x^2} \to 0 + \frac{1}{x^4} \to 0 - \frac{2}{x^5} \to 0}{3+4/x^3+1/x^5} = \frac{0}{3} = 0$$

3. (12 pts) Use the **definition of the derivative** to find $f'(x)$ for $f(x)=x^2+3x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2+3(x+h)-[x^2+3x]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{3x}+3h-\cancel{x^2}-\cancel{3x}}{h} = \lim_{h \to 0} \frac{2xh+h^2+3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+3+h)}{\cancel{h}} = \lim_{h \to 0} 2x+3+h = 2x+3$$

$f'(x) =$ $2x+3$

4. (6 pts each) Suppose the total cost of producing x bicycles is given by $C(x) = 5000 + 40x + 0.5x^2$ .

(a) How fast is the Cost changing with respect to the number of bicycles produced?

$C'(x) = 40 + 2(0.5)x = 40 + x$

Answer: $40 + x$

(b) What is the rate of change of the Cost function when 10 bicycles are produced?

$C'(10) = 40 + 10 = 50$

Answer: 50

5. (5 pts each) Find $y'$ for the following functions. **(Do NOT bother to simplify!!!)**

(a) $y = \sqrt[3]{x} - \frac{5}{2x^2} + 4x - 7x^{-1} = x^{1/3} - \frac{5}{2}x^{-2} - 7x^{-1}$

$y' = \frac{1}{3}x^{-\frac{2}{3}} - (-2)\frac{5}{2}x^{-3} - (-1)7x^{-2} + 4$ ← from deriv. of $4x$

$y' = \frac{1}{3}x^{-2/3} + 5x^{-3} + 7x^{-2} + 4$

(b) $y = (4x^4 + x^2)\left(5x + \frac{1}{x^{2/3}}\right)$ $y' = (4x^4 + x^2)'(5x + \frac{1}{x^{2/3}}) + (4x^4 + x^2)(5x + x^{-\frac{2}{3}})'$

$= (16x^3 + 2x)(5x + 1/x^{2/3}) + (4x^4 + x^2)(5 - \frac{2}{3}x^{-5/3})$

$y' = (16x^3 + 2x)(5x + \frac{1}{x^{2/3}}) + (4x^4 + x^2)(5 - \frac{2}{3}x^{-\frac{5}{3}})$

(Note: This is #5 continued.)
Find $y'$ for the following functions. **(Do NOT bother to simplify!!!)**

(c) $y=\dfrac{2x^3-1}{5x^3}$

$$y' = \frac{(2x^3-1)'\,5x^3 - (2x^3-1)(5x^3)'}{(5x^3)^2} = \frac{6x^2\cdot 5x^3 - (2x^3-1)15x^2}{25x^6}$$

$$y' = \frac{6x^2\cdot 5x^3 - (2x^3-1)15x^2}{25x^6}$$ ← could leave as $(5x^3)^2$

(d) $y=\sqrt{5x+3x^3} = (5x+3x^3)^{1/2}$

$$y' = \tfrac{1}{2}(5x+3x^3)^{-\frac{1}{2}}\cdot(5x+3x^3)' = \tfrac{1}{2}(5x+3x^3)^{-1/2}(5+9x^2)$$

$$y' = \tfrac{1}{2}(5x+3x^3)^{-1/2}\cdot(5+9x^2)$$

~~6. (10 pts) Find the second derivative of~~ $\cancel{f(x) = \frac{2}{3x+1}}$.

second derivative: ______________________

7. (16 pts) For $f(x)=3x(x^2-4x)$ , find the equation of the tangent line to the curve at x = -1.

Slope is $f'(-1)$.

$f'(x) = 3(x^2-4x) + 3x(x^2-4x)' = 3(x^2-4x) + 3x(2x-4)$

So $m = f'(-1) = 3((-1)^2-4(-1)) + 3(-1)(2(-1)-4)$

$= 3(1+4) - 3(-6) = 15 + 18 = 33$

When $x=-1$, $y = f(-1) = -3((-1)^2+4) = -3 \cdot 5 = -15$

Point slope equation

$y - (-15) = 33(x-(-1))$ or $y + 15 = 33(x+1)$

Tangent Line: $y + 15 = 33(x+1)$