# Solutions to Review Problems for Final Exam

1. a. $\lim_{x \to 2} \frac{x^3 - 2x^2}{x^2 + x - 6}$ if plug in $x = 2$ get $\frac{8 - 2 \cdot 4}{4 + 2 - 6} = \frac{0}{0}$

so limit might exist. Factor!

$$\lim_{x \to 2} \frac{x^2 \cancel{(x-2)}}{\cancel{(x-2)}(x+3)} = \lim_{x \to 2} \frac{x^2}{x+3} = \frac{2^2}{2+3} = \boxed{\frac{4}{5}}$$

b. $\lim_{x \to \infty} \frac{e x^3 + x + 1}{5x^3 + x^2 + 5} = \lim_{x \to \infty} \frac{e x^3}{5x^3} = \lim_{x \to \infty} \frac{e}{5} = \boxed{\frac{e}{5}}$

(note: $e$ is just a number).

2. a. $f(x) = \frac{2x^3 + 2x + 6}{x^2 - 1}$ Quotient Rule

$$f'(x) = \frac{(6x^2 + 2)(x^2 - 1) - 2x(2x^3 + 2x + 6)}{(x^2 - 1)^2}$$

b. $f(x) = (x^2 + 1) \ln(x^3 - 1)$ Prod. Rule

$$f'(x) = 2x \ln(x^3 - 1) + (x^2 + 1)[\ln(x^3 - 1)]'$$ Chain Rule

$$= 2x \ln(x^3 - 1) + (x^2 + 1) \frac{1}{x^3 - 1} (x^3 - 1)'$$

$$= \boxed{2x \ln(x^3 - 1) + (x^2 + 1) \frac{1}{x^3 - 1} \cdot 3x^2}$$

c. $f(x) = e^{5x^2} + \frac{3}{\sqrt{x}} - 2x^5 = e^{5x^2} + 3x^{-\frac{1}{2}} - 2x^5$

$f'(x) = e^{5x^2}(5x^2)' + (-\frac{1}{2})3x^{-3/2} - 10x^4$

$= \boxed{e^{5x^2}10x - \frac{3}{2}x^{-3/2} - 10x^4}$

4. $f(x) = x + e^{-x}$

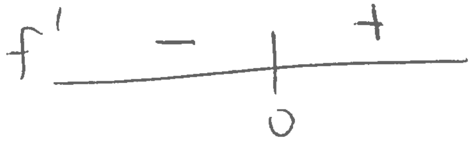
a. $f'(x) = 1 - e^{-x}$

$f'(x) = 1 - e^{-x} = 0 \Rightarrow e^{-x} = 1 \Rightarrow \frac{1}{e^x} = 1$ or $e^x = 1$

$e^x = 1 \Rightarrow \boxed{x = 0}$ only critical value.

$x < 0$ say $x = -1$ $f'(-1) = 1 - e^1 = 1 - e < 0$ f decreasing

$x > 0$ say $x = 1$ $f'(1) = 1 - e^{-1} = 1 - \frac{1}{e} > 0$ f inc.



f increasing on $(0, \infty)$

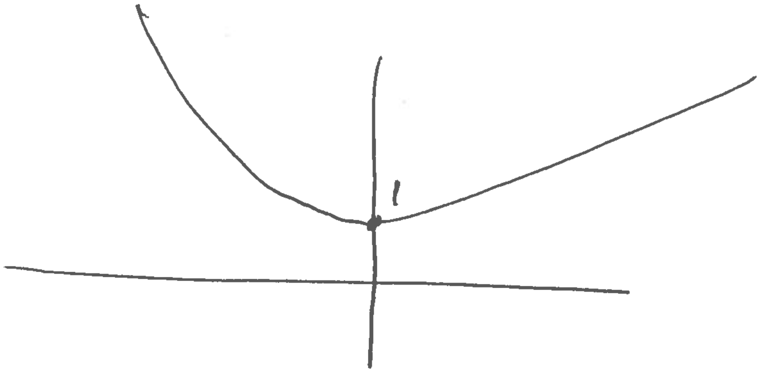
f decreasing on $(-\infty, 0)$

$\boxed{(0, f(0)) = (0, 1) \text{ is rel. min. point}}$

b. $f''(x) = (1 - e^{-x})' = -(e^{-x})' = -(-e^{-x}) = e^{-x} > 0$ all x

so $\boxed{f \text{ is concave up on } (-\infty, \infty)}$

c.



Concave up
everywhere
rel. min at (0,1).

3. $f(x) = x^2 - 2x$

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - [x^2 - 2x]}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{x^2} + \cancel{2x}}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 - 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x - 2 + h)}{\cancel{h}}$

$= \lim_{h \to 0} 2x - 2 + h = \boxed{2x - 2}$

b. Slope $m = f'(1) = 2 \cdot 1 - 2 = 0$

$f(1) = 1 - 2 = -1$

$y + 1 = m(x - 1) = 0$

$\boxed{y + 1 = 0}$

5. $2x^3 - 5xy + y^2 = 4$

$6x^2 - 5y - 5xy' + 2yy' = 0$

$y'(2y - 5x) = 5y - 6x^2 \Rightarrow y' = \frac{5y - 6x^2}{2y - 5x}$

6. $R(x) = 100x - 0.2x^2$

a. $R'(x) = 100 - 2 \cdot (0.2)x = 100 - 0.4x = 0$

(Finding critical values) $0.4x = 100$

$$x = \frac{100}{0.4} = 250.$$

Since $R'' = -0.4$, the graph is concave down

$\Rightarrow$ R has a maximum at $\boxed{x = 250}$

b. Max. Revenue $= R(250) = 100 \cdot 250 - 0.2(250)^2$

$= 25000 - 0.2 \cdot 62{,}500$

$= 25{,}000 - 12{,}500$

$= \boxed{12{,}500}$

c. Must maximize $R(x)$ on interval $0 \leq x \leq 200$.

The only critical value is $x = 250$ which is not in the interval, so the max. must occur at an endpoint

Since $R(0) = 0$, $R(200) = 100 \cdot 200 - 0.2(200)^2$

$= 20{,}000 - 0.2 \cdot 40000$

$= 20{,}000 - 8{,}000 = 12{,}000$

so $\boxed{\text{200 units will maximize Revenue}}$

7. a. $\int_1^2 \left(10x^4 - \frac{12}{x^3}\right) dx = \int_1^2 10x^4 - 12x^{-3}\, dx$

$= 10\int_1^2 x^4\, dx - 12\int_1^2 x^{-3}\, dx$

$= 10\frac{x^5}{5}\Big|_1^2 - 12\frac{x^{-2}}{-2}\Big|_1^2$

$= 10\cdot\left(\frac{2^5}{5} - \frac{1}{5}\right) + 6\left(2^{-2}-1\right) = 2\cdot 31 + 6\left(-3/4\right) = 62 - \frac{9}{2} = \boxed{\frac{115}{2}}$

b. $\int \frac{-2}{5x+7}\, dx$ substitute $u = 5x+7$

$du = 5\, dx$

$dx = \frac{1}{5}\, du$

$\int \frac{-2\cdot\frac{1}{5}\, du}{u} = -\frac{2}{5}\int \frac{1}{u}\, du$

$= -\frac{2}{5}\ln|u| + C = \boxed{-\frac{2}{5}\ln|5x+7| + C}$

c. $\int 4x^3 e^{-x^4+1}\, dx$. Substitute $u = -x^4+1$

$du = -4x^3\, dx$

$x^3\, dx = -\frac{1}{4}\, du$

$\int 4e^u\left(-\frac{1}{4}\right)du = -\int e^u\, du$

$= -e^u + C$

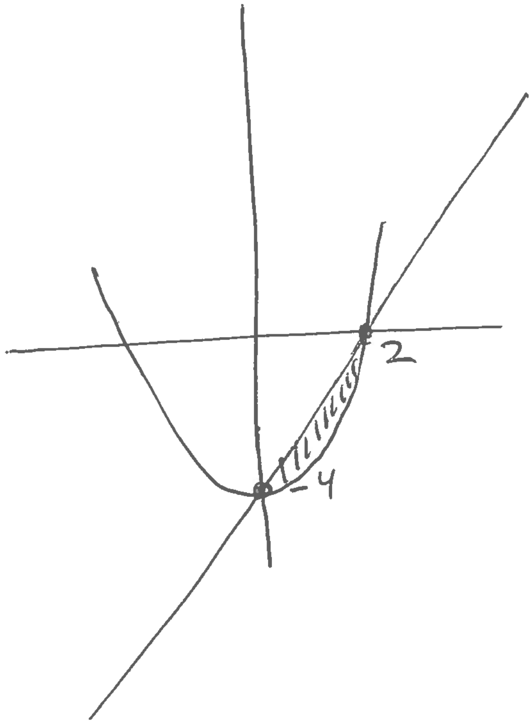
$= \boxed{-e^{-x^4+1} + C}$

⑧ $y = 2x-4, \quad y = x^2-4$

$y = 2x-4$

$x=0 \quad y=-4$

$y=0, x=2$



Intersectin points:

$2x - \cancel{4} = x^2 - \cancel{4}$

$2x = x^2$

$x^2 - 2x = 0$

$x(x-2) = 0$

$\boxed{x=0, \quad x=2}$

$$A = \int_0^2 (2x-4)-(x^2-4)\,dx$$

$$= \int_0^2 2x-\cancel{4}-x^2+\cancel{4}\,dx = \int_0^2 2x-x^2\,dx = x^2-\frac{x^3}{3}\Big|_0^2$$

$$= 2^2-\frac{2^3}{3}-(0-0) = 4-\frac{8}{3} = \boxed{\frac{4}{3}}.$$

9. $P(t) = 100 - 0.2t + 0.3t^2$

Average value of P from $t=0$ to $t=10$!

$$\bar{P} = \frac{1}{10}\int_0^{10} P(t)\,dt = \frac{1}{10}\int_0^{10} 100 - 0.2t + 0.3t^2\,dt$$

$$= \frac{1}{10}\left(100t - 0.2\frac{t^2}{2} + 0.3\frac{t^3}{3}\right)\Big|_0^{10}$$

$$= \frac{1}{10}\left(100t - 0.1t^2 + 0.1t^3\right)\Big|_0^{10}$$

$$= \frac{1}{10}\left[100\cdot 10 - 0.1\cdot 10^2 + 0.1\cdot 10^3\right]$$

$$= \frac{1}{10}\left(1,000 - 10 + 100\right)$$

$$= \frac{1090}{10} = \boxed{109}$$

10. $C'(x) = 6x+60, \quad R'(x) = 180 - 2x$

$C(10) = 1,000.$

a. First we find $C(x)$ and $R(x)$

$$C(x) = \int C'(x)\,dx = \int 6x+60\,dx = \frac{6x^2}{2} + 60x + C = 3x^2+60x+C$$

$$C(10) = 1,000 = 3\cdot 10^2 + 60\cdot 10 + C$$

$$1,000 = 900 + C \Rightarrow C = 100$$

$$C(x) = 3x^2+60x+100$$

$R(x) = \int R'(x)\,dx = \int 180 - 2x\,dx = 180x - x^2 + C$

$R(0) = 0 = C$ ($x = 0 \Rightarrow$ revenue is zero).

$R(x) = 180x - x^2.$

$P(x) = R(x) - C(x) = 180x - x^2 - (3x^2 + 60x + 100)$

$$\boxed{P(x) = 120x - 4x^2 - 100}$$

b. Find critical value :

$P'(x) = 120 - 8x = 0$

$8x = 120$

$x = \frac{120}{8} = 15.$

Since $P''(x) = -8 < 0$, $P$ concave down and $P$ has a maximum at $\boxed{x = 15}$

c. $P(15) = 120 \cdot 15 - 4 \cdot (15)^2 - 100$

$= 1800 - 900 - 100$

$= \boxed{800}$ profit.