Solutions

MATH 1100-001

EXAM 2

Instructions:. Put your name on the exam. Show your work for full credit. Calculators and other electronic devices (including cell phones) are not allowed. You are allowed one side of an 8.5 by 11 inch sheet of notes.

1. Suppose that the the total cost function is $C(x) = 1000 + 92x$, where x is the number of goods produced, and that price per unit is $p(x) = 100 + x - \frac{x^2}{3}$ (the price at which x goods will sell).

a. (5 points) Find the revenue function.

$$R(x) = x\,p(x) = x\left(100 + x - \frac{x^2}{3}\right) = 100x + x^2 - \frac{x^3}{3}$$

b. (5 points) Find the marginal revenue, and marginal cost functions.

$R'(x) = 100 + 2x - x^2$ = Marginal revenue function

$C'(x) = (100 + 92x)' = 92$ = Marginal cost function.

c. (5 points) Find the profit function.

$$P(x) = R(x) - C(x) = 100x + x^2 - \frac{x^3}{3} - (100 + 92x)$$

d. (5 points) Find the marginal profit function.

$$P'(x) = R'(x) - C'(x) = 100 + 2x - x^2 - 92$$

$$= 8 + 2x - x^2$$

1e. (10 points) Find the number of units sold that maximizes the profit.

$P'(x) = 0 \Rightarrow 8 + 2x - x^2 = 0 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$

positive value of x: $\boxed{x = 4}$

$P''(x) = 2 - 2x$, $P''(4) = 2 - 8 < 0 \Rightarrow$ maximum at $x = 4$

2nd deriv. test (could use 1st deriv. test)

2. Suppose that the cost for producing x goods is $C(x) = 40 + 0.1x^2$

a. (5 points) Find $\overline{C}(x)$, the average cost per unit.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{40 + 0.1x^2}{x} = \frac{40}{x} + \frac{0.1x^2}{x} = \frac{40}{x} + 0.1x$$

b. (20 points) Find the number of units produced that minimizes the average cost per unit. What is the minimum average cost per unit?

Critical values: $\overline{C}'(x) = (0.1x + 40x^{-1})' = 0.1 - 40x^{-2} = 0.1 - \frac{40}{x^2}$

$C'(x) = 0 \Rightarrow 0.1 - \frac{40}{x^2} = 0$, $0.1 = \frac{40}{x^2}$, $x^2 = \frac{40}{0.1} = 400$

$x = \pm\sqrt{400}$, $x = \pm 20$. $x > 0 \Rightarrow x = 20$.

1st derivative test: for $x < 20$, say $x = 10$ $\overline{C}'(10) = 0.1 - \frac{40}{100} = -0.3 < 0$

for $x > 20$, say $x = 30$ $\overline{C}'(30) = 0.1 - \frac{40}{900} > 0$

$\overline{C}'$ $-$ $+$

20

$\Rightarrow$ $\overline{C}$ has a minimum at $\boxed{x = 20.}$

The average minimum cost is

$\overline{C}(20) = \frac{40}{20} + 0.1 \cdot (20) = 2 + 2 = \boxed{4}$

3. Let $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 2$.

a. (5 points) Find the critical values of $f(x)$.

$$f'(x) = 3 \cdot \frac{1}{3} x^2 - 2 \cdot 2x + 3 = x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$\Rightarrow \boxed{x = 1, 3}$$

b. (5 points) Find the intervals on which f is increasing and decreasing.

$x < 1$ say $x = 0$ $f'(0) = 3 > 0$

$1 < x < 3$ say $x = 2$ $f'(2) = 4 - 8 + 3 = -1 < 0$

$3 < x$ say $x = 4$ $f'(4) = 16 - 16 + 3 = 3 > 0$

$\Rightarrow$ f is increasing on $(-\infty, 1)$ and $(3, \infty)$
f is decreasing on $(1, 3)$

f': $+$ | $-$ | $+$

1 3

c. (10 points) Find the relative maximum and minimum points (if any).

From the 1st derivative test f has a rel. max at 1, rel. min at 3

$f(1) = \frac{1}{3} - 2 + 3 + 2 = 10/3$ $\boxed{(1, 10/3) \text{ rel. max point}}$

$f(3) = \frac{27}{3} - 2 \cdot 9 + 9 + 2 = 2$ $\boxed{(3, 2) \text{ rel. min. point}}$

d. (10 points) Find the intervals on which f is concave up and concave done, as well as any inflection points.

$f''(x) = 2x - 4$. $f''(x) = 0 \Rightarrow 2x - 4 = 0 \Rightarrow x = 2$.

$x < 2$, say $x = 0$ $f''(0) = -4 < 0$

$x > 2$ say $x = 3$ $f''(3) = 6 - 4 = 2 > 0$

f'': $-$ | $+$

2

$\boxed{f \text{ concave up on } (2, \infty)}$

$\boxed{f \text{ concave down on } (-\infty, 2)}$

4. Let f be the function:

$$f(x) = \frac{2x^3 + 1}{x(x^2 - 1)}$$

a. (5 points) find the domain of f.

Domain: all x where $f(x)$ is defined. $f(x)$ not defined only when denominator is zero: $x(x^2-1)=0 \Rightarrow x(x+1)(x-1)=0$

Domain all x except $x = 0, -1, 1$.

b. (5 points) Find the vertical asymptotes (if any).

Verticle asymptotes occur at values where denominator is zero; if numerator not zero. $2x^3+1 \neq 0$ when $x = -1, 0, 1$ so verticle asymptotes are:

$$x = -1$$
$$x = 0$$
$$x = 1$$

c. (5 points) Find the horizontal asymptotes (if any).

Horizontal asymptote are given by $y = \lim_{x\to\infty} \frac{2x^3+1}{x(x^2-1)}$

$$= \lim_{x\to\infty} \frac{2x^3+1}{x^3-1} = \lim_{x\to\infty} \frac{2x^3}{x^3} = 2$$

$$\Rightarrow \boxed{y = 2}$$