Commutative Algebra and its Interactions
A conference in honor of Mel Hochster

	July 31 *Thursday*	**August 1** *Friday*	**August 2** *Saturday*	**August 3** *Sunday*	**August 4** *Monday*	**August 5** *Tuesday*
10:00–11:00	**Eisenbud** *1360 EH*	**Bruns** *1324 EH*	**Roberts** *1360 EH*	**Huneke** *1360 EH*	**Lyubeznik** *1360 EH*	**Avramov** *1360 EH*
11:20–12:00	**Dao** *1360 EH*	**Lakshmibai** *1324 EH*	**Dutta** *1360 EH*	**Brenner** *1360 EH*	**Blickle** *1360 EH*	**Aberbach** *1360 EH*
	Lunch	*Lunch*	*Lunch*	*Lunch*	*Lunch*	✈
2:00–2:40	**Heitmann** *1360 EH*	**Lipman** *1324 EH*	**Miller** *1360 EH*	**Mustaţă** *B844 EH*	**Schwede** *1200 CHEM*	
3:00–3:40	**Yao** *1360 EH*	**Swanson** *1324 EH*	**Enescu** *1360 EH*	**Takagi** *B844 EH*	**Hara** *1200 CHEM*	
		Tea		*Tea*	*Tea*	
4:20–5:00		**Polini** *1324 EH*			**Vraciu** *1200 CHEM*	

Coffee: Each morning.
East Hall Atrium, 9:30–10:00am.

Reception: July 31, Thursday.
East Hall Atrium, starting 4:15pm.

Banquet: August 2, Saturday.
Great Lakes North and Central rooms of Palmer Commons, 6:00–9:00pm.

Bridge: August 3, Sunday.
On the River Kwai, starting 4:00pm.

Ian Aberbach, *Estimating lower bounds for Hilbert-Kunz multiplicities*

Let (R, m) be an excellent local ring of dimension d. If R has no components of dimension less than d, then R is regular if and only if the Hilbert-Kunz multiplicity is equal to 1. Actually computing Hilbert-Kunz multiplicities is quite difficult, but in joint work with Florian Enescu, we have shown that there is a lower bound (strictly greater than 1) for the Hilbert-Kunz multiplicity of such a non-regular ring. This talk, which will coordinate with the earlier talk by Enescu, will elaborate on some of the methods for making these estimates.

Luchezar Avramov, *New avatars of Intersection Theorems*

The classical intersection theorems in commutative algebra deal with modules or with complexes of modules over commutative rings. In recent joint work with Ragnar-Olaf Buchweitz, Srikanth Iyengar, and Claudia Miller, similar results have been shown to hold for differential modules, that is, modules equipped with a square-zero endomorphism. At present they are restricted to algebras over a field, or to low dimensions, as Mel Hochster's maximal Cohen-Macaulay modules play an essential role at one step of the proof.

Manuel Blickle, *Modules with a Frobenius action*

The action of the Frobenius on various objects is the main tools in positive characteristic algebra. In this talk I will recall several notions of modules with a Frobenius action which were used in our field with great success. Then I report on some recent joint work with Böckle which relates these different notions with each other via a Grothendieck-Serre duality which is enriched with an action of the Frobenius. A key feature is that three of Grothendiecks six operations are preserved under this equivalence. An applications to local invariants will also be outlined.

Holger Brenner, *Tight closure is the better idea(l)*

Generic degree bounds for inclusion to tight closure. One of the standard features of tight closure theory is that theorems for regular rings, which express a containment to an ideal, can be generalized to arbitrary rings if we replace the ideal by its tight closure. The Briançon-Skoda theorem is a typical example. In this talk we are interested in standard-graded rings and the situation where a degree tuple is fixed. What elements of what degree do belong to the tight closure of an ideal generated by generically chosen elements of this degree tuple. This question is non trivial for polynomial rings, where it is directly related to the Fröberg conjecture, which is known in dimension three. We show how generic degree bounds on the polynomial ring imply generic degree bounds for tight closure in an arbitrary standard-graded ring. Surprisingly or not, the degree bounds do not depend on the rings, showing that tight closure of generic ideals behaves nicer that ideals themselves. This is joint work with Helena Fischbacher-Weitz.

Winfried Bruns, *Rings of invariants*

A large part of Hochster's work in the 70s was devoted to proving the Cohen-Macaulay property for rings of invariants of linearly reductive groups. After several special cases (determinantal rings, Grassmannians, torus invariants) had been completed, the general case was settled by the Hochster-Roberts theorem. It uses reduction to characteristic p, another leitmotif in Hochster's work that has found its strongest expression in tight closure theory. We will survey Hochster's work and relate it to newer developments.

Hailong Dao, *Vanishing of Tor and torsion-freeness of the divisor class groups*

We show that if $R = S/(f)$ is a local hypersurface of an unramified regular local ring S, such that $\dim R = 3$ and R has isolated singularity, then the class group of R is torsion-free. This work is motivated by a conjecture of Gabber stating that for any local complete intersection R of dimension 3, the Picard group of the punctured spectrum of R is torsion-free. The proof use a function on the Grothendieck group of R defined by Hochster.

Sankar Dutta, *Filtered Resolution, Intersection Multiplicity and Blow-up*

We would like to present an approach for studying Serre's conjecture on intersection multiplicity in the light of intersection multiplicities of the tangent cone and of the blow-up of the regular local ring at its closed point.

David Eisenbud, *Free Resolutions*

I will survey some of Hochster's work related to free resolutions, and also some of the recent work related to it.

Florian Enescu, *Hilbert-Kunz multiplicities*

We will present a series of inequalities that relate the Hilbert-Kunz multiplicity of a local ring R to other various remarkable invariants of R. These inequalities will be used to develop lower bounds, strictly greater than 1, for the Hilbert-Kunz multiplicity of a nonregular local ring. Applications to the Watanabe-Yoshida conjecture on the minimal Hilbert-Kunz multiplicity on a nonregular ring in dimensions 5 and 6 will be provided, as well as for rings of small Hilbert-Samuel multiplicity. This is joint work with Ian Aberbach.

Nobuo Hara, *Strong F-regularity vs. log terminal singularity in non-$\mathbb{Q}$-Gorenstein case*

It is known that a $\mathbb{Q}$-Gorenstein ring of characteristic zero has F-regular type if and only if it has only log terminal singularities. According to a formulation due to De Fernex and Hacon, I will discuss a generalization of this correspondence to non-$\mathbb{Q}$-Gorenstein rings. At the present moment the necessity still requires some finiteness condition that is weaker than $\mathbb{Q}$-Gorensteinness.

Raymond Heitmann, *Lifting Seminormality*

It has long been known that if R is a local Noetherian ring and y is a regular element in the maximal ideal, then R/yR normal forces R to be normal as well. It is shown that the same is true for the property of seminormality. Recall that a ring is seminormal if whenever x is an element of the total quotient ring such that x^2 and x^3 are in the ring, so is x.

Craig Huneke, *The work of Mel Hochster, a personal view*

This talk will focus on Hochster's work and its profound influence on commutative algebra. The talk will be largely personal reflections on his work both past and present.

V. Lakshmibai, *Frobenius-split property for certain rings of invariants*

For the actions of $SL(n)$, $SO(n)$ on certain algebraic varieties (analogous to the actions of $GL(n)$, $O(n)$ appearing in classical invariant theory), we first show the Cohen-Macaulayness for the corresponding rings of invariants, and then show the Frobenius-split properties for these rings of invariants.

Joseph Lipman, *Grothendieck operations and coherence in categories*

I will illustrate the yoga of Grothendieck Duality, in the scaled-down context of modules over rings and quasi-finite ring homomorphisms. The emphasis will be on basic category-theoretic properties of familiar maps, thought of as relations among Grothendieck operations (tensor, hom, restriction and extension of scalars); and on the need to deduce commutativity of many natural diagrams from these "axiomatic" properties. (The problem, purely formal, is no different for full-blown Grothendieck Duality, in the context of derived categories over noetherian schemes and separated finite-type scheme-maps.) The resulting overwhelming tedium issues a challenge: fight back with some form of automated reasoning, or better, with general theorems. This is the stuff of coherence in categories.

Gennady Lyubeznik, *Frobenius and Local Cohomology*

Local cohomology is an integral part of commutative algebra and the Frobenius morphism in characteristic p is a powerful tool in studying local cohomology. Both have been extensively used in Mel's work. We will review some of the highlights of the applications of the Frobenius to local cohomology.

Claudia Miller, *Concerning limit Hilbert-Kunz multiplicities*

In joint work with Holger Brenner and Jinjia Li, we investigate cases when a more naïve limit turns out to give the limit Hilbert Kunz multiplicity. The main case is that of ideals in the affine cone of smooth nonsingular curves; another case is that of maximal ideals in diagonal hypersurfaces.

Mircea Mustaţă, *Bernstein-Sato polynomials in positive characteristic*

I will discuss a definition of Bernstein-Sato polynomials in positive characteristic, motivated by the connection with the V-filtration in characteristic zero. This notion turns out to be related to the jumping numbers for the generalized test ideals of Hara and Yoshida.

Claudia Polini, *Blowup algebras of codimension two ideals*

Paul Roberts, *The Homological Conjectures*

One of Mel Hochster's many contributions to the field of Commutative Algebra was the formulation of a group of questions that became known as the Homological Conjectures. In this talk we will discuss these conjectures, including their history, progress that has been made on them over the years, and recent advances in the subject.

Karl Schwede, *A geometric characterization of F-ideals/centers of F-purity*

In this talk we will discuss a positive characteristic analogue of centers of log canonicity (a notion from algebraic geometry). We will explore the structure of these objects in the F-pure setting. We will also link this notion to F-ideals, which are annihilators of Frobenius stable submodules of the top local cohomology (and to generalizations of such objects as studied by Lyubeznik and Smith).

Irena Swanson, *Integral closure*

This is joint work with Anurag Singh. I will present the history of the computation of integral closures, starting with Dedekind's determination of the integral closures of cyclic extensions of the ring of integers, and ending with our recent joint work. I will concentrate mostly on the algorithmic aspects of the computation. The first algorithmic consideration is due to Stolzenberg from 1975, and was improved by Seidenberg. A more effective method for computing the integral closure of affine domains is due to Grauert, Remmert, and de Jong, and further modifications and refinements are due to Vasconcelos. These algorithms successively approximate the integral closure from below, namely by building successively strictly larger rings between the original ring and its integral closure. Based on a specialized 2003 algorithm of Leonard-Pellikaan, we prove a more general version of the construction of the integral closure that starts instead with a finitely generated module over the ring that contains the integral closure, and the successive steps produce strictly smaller submodules that contain the integral closure.

Shunsuke Takagi, *Computations of log canonical thresholds: a positive characteristic approach*

The log canonical threshold is an invariant of singularities which plays an important role in higher-dimensional birational geometry. It is, however, not easy to compute this invariant in general. I will talk about how to compute the log canonical threshold of a certain binomial ideal using characteristic p methods. This is a joint work with Takafumi Shibuta.

Adela Vraciu, *Drops in Joint Hilbert-Kunz multiplicity*

We introduce a new version of $\mathfrak{a}$-tight closure, related to the one introduced by Hara and Yoshida in 2000, but which has the advantage that it is a true closure. We study a family of such closures, obtained by allowing a positive real number t as exponent. These closures get larger as the parameter t increases. The Joint Hilbert-Kunz multiplicity provides a criterion for membership in the $\mathfrak{a}$-tight closure (similar to the one proved by Hochster and Huneke for tight closure, using Hilbert-Kunz multiplicities). We study the difference between the Joint Hilbert-Kunz multiplicities of two ideals, as a function of the parameter t.

Yongwei Yao, *Embedding theorems and uniform test exponents*

Assume R is a commutative Noetherian ring and all modules are finitely generated. Then the embedding theorem states that every R-module of finite projective dimension embeds into a finite direct sum of cyclic R-modules each of which is the quotient of R by an ideal generated by an R-regular sequence. In fact, this embedding theorem applies to all R-modules of finite G-dimension. Further assume R is a domain of prime characteristic. Then, under mild conditions (e.g., R is essentially of finite type over a complete local ring or an F-finite homomorphic image of a Gorenstein ring), there exists a (fixed) module-finite extension domain S of R with the following property: For every R-module of finitely phantom projective dimension, its scalar extension to S *weakly* embeds into a finite direct sum of cyclic S-modules each of which is the quotient of S by a parameter ideal. Here a weak embedding means a linear map whose kernel is contained in the tight closure of 0. As an application of the above weak embedding theorem, we can show the existence of (uniform) test exponents for all modules of finite phantom projective dimension. All the above results are joint work with Mel Hochster. Time permitting, I will also talk about the existence of (uniform) test exponents for all R-modules if R has finite F-representation type.