1. The number 0-251-32133-6 is obtained by transposing two consecutive digits in a valid ISBN number; find the possible valid ISBN numbers.

2. Solve $563x \equiv 7943 \mod 18965$ using your favorite calculator or software.

3. Is 2 a primitive root modulo 103? Try to do this efficiently!

4. Using the Chinese remainder theorem, find all solutions of $x^2 = 2$ in the ring $\mathbb{Z}/119$. (Note that $119 = 7 \times 17$.)

5. Determine the inverse of $7 - 3\sqrt{5}$ modulo 17.

6. Consider the field $\mathbb{F}_{25} := \mathbb{Z}[\sqrt{2}] / 5$. Determine the elements of $\mathbb{F}_{25}^\times$ of norm 1. List the order of these elements.

7. (a) Determine if 80 is a square modulo 127.
   (b) Determine if 208 is a square modulo 485.

8. How many solutions does $x^2 = 7$ have in each of the following rings? (You need not find the solutions.)
   (a) $\mathbb{Z}/19$
   (b) $\mathbb{Z}/21$
   (c) $\mathbb{Z}/57$

9. Let $p$ be an odd prime. Fill in the table:

<table>
<thead>
<tr>
<th></th>
<th>$p \equiv 1 \mod 8$</th>
<th>$p \equiv 3 \mod 8$</th>
<th>$p \equiv 5 \mod 8$</th>
<th>$p \equiv 7 \mod 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left( \frac{-1}{p} \right)$</td>
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<tr>
<td>$\left( \frac{2}{p} \right)$</td>
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<td>$\left( \frac{-2}{p} \right)$</td>
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</table>

Prove that $x^8 \equiv 16 \mod p$ has a solution.

10. Let $p$ be a prime integer. Show that $1^n + 2^n + \cdots + (p-1)^n$ is congruent to 0 or $-1 \mod p$, depending on $n$. 