Solution Key / out of 85 pts

Check whether they have attempted every excersize.
Check thoroughly the ones solved here.

Exercizes in the solution key - 10 points each
Exercizes not in the solution key - 5 points each (just check whether they have them

sec. 1.1.

(1) $\frac{\partial u_1}{\partial x} + \frac{\partial u_1}{\partial t} = 0$

$\frac{\partial u_2}{\partial x} + \frac{\partial u_2}{\partial t} = 0$

$u = c_1 u_1 + c_2 u_2$

$\frac{\partial u}{\partial x} = c_1 \frac{\partial u_1}{\partial x} + c_2 \frac{\partial u_2}{\partial x}$

$\frac{\partial u}{\partial t} = c_1 \frac{\partial u_1}{\partial t} + c_2 \frac{\partial u_2}{\partial t}$

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = c_1 \frac{\partial u_1}{\partial x} + c_2 \frac{\partial u_2}{\partial x} + c_1 \frac{\partial u_1}{\partial t} + c_2 \frac{\partial u_2}{\partial t} =$

$= c_1 \underbrace{\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_1}{\partial t} \right)}_{= 0} + c_2 \underbrace{\left(\frac{\partial u_2}{\partial x} + \frac{\partial u_2}{\partial t} \right)}_{= 0} = 0$

sec. 1.1

1	- 10 points
5	5 pts.
6	5 pts
10	10 pts
11	10 pts
	40 pts

(10)

a) The directional derivative is zero in direction (a, b)

b) $a \dfrac{\partial u}{\partial x} + b \dfrac{\partial u}{\partial y} = 0$

$$\frac{\partial u}{\partial x} + \frac{b}{a} \frac{\partial u}{\partial y} = 0$$

$$\frac{dy}{dx} = \frac{b}{a}$$

$$y = \frac{b}{a} x + c \Rightarrow c = y - \frac{b}{a} x \quad \text{characteristics}$$

c) $u = f\left(\cancel{b}\, y - \frac{b}{a} x \right)$

(11) a) $\dfrac{\partial u}{\partial x} + x^2 \dfrac{\partial u}{\partial y} = 0$

$$\frac{dy}{dx} = x^2$$

$$y = \frac{1}{3} x^3 + c \Rightarrow c = y - \frac{1}{3} x^3$$

$$u = f\left(y - \frac{1}{3} x^3\right)$$

b) $$\left.\begin{aligned} \frac{\partial u}{\partial x} &= f'\left(y - \tfrac{1}{3}x^3\right)\left(-x^2\right) \\ \frac{\partial u}{\partial y} &= f'\left(y - \tfrac{1}{3}x^3\right) \end{aligned}\right\} \Rightarrow \frac{\partial u}{\partial x} + x^2 \frac{\partial u}{\partial y} = 0$$

Sec. 1.2

3	– 10 pts
5a	– 10 pts
15	– 5 pts
16	10 pts
18	5 pts
19	5 pts
	45 pts

(3) $u(x,t) = F(x+ct) + G(x-ct)$

$$\frac{\partial^2 u}{\partial x^2} = F''(x+ct) + G''(x-ct)$$

$$\frac{\partial^2 u}{\partial t^2} = F''(x+ct)(c^2) + G''(x-ct)(c^2)$$

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(5a)

$$u(x,t) = \frac{1}{2(1+(x+ct)^2)} + \frac{1}{2(1+(x-ct)^2)}$$

$$u(x,0) = \frac{1}{2(1+x^2)} + \frac{1}{2(1+x^2)} = \frac{1}{1+x^2}$$

$$\frac{\partial u}{\partial x}(x,t) = -\frac{1}{2}\frac{2(x+ct)\,c}{(1+(x+ct)^2)^2} - \frac{1}{2}\frac{2(x-ct)(-c)}{(1+(x-ct)^2)^2}$$

$$= -\frac{c(x+ct)}{(1+(x+ct)^2)^2} + \frac{c(x-ct)}{(1+(x-ct)^2)^2}$$

$$\frac{\partial u}{\partial x}(x,0) = -\frac{cx}{(1+x^2)^2} + \frac{cx}{(1+x^2)^2} = 0$$

$u = F(x+ct) + G(x-ct)$

$u(x,0) = F(x) + G(x) = \frac{1}{1+x^2}$

$\frac{\partial u}{\partial t} = cF' - cG'$

$\frac{\partial u}{\partial t}(x,0) = cF' - cG'$

$\Rightarrow F' = G'$

$\int F'(s)\,ds = \int G'(s)\,ds$

$\Rightarrow F(s) = G(s) + C$

$\Rightarrow F(x) = G(x) + C$

But

$F(x) + G(x) =$

$= 2G(x) + C = \frac{1}{1+x^2}$

$\Rightarrow C = 0 \quad G(x) = F(x) =$

$= \frac{1}{2}\left[\frac{1}{\ldots}\right]$

(16)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L}\cos\frac{n\pi ct}{L} + b_n \sin\frac{n\pi x}{L}\sin\frac{n\pi ct}{L}$$

$$\frac{\partial u}{\partial t} = \sum_{n=1}^{\infty} -a_n \frac{n\pi c}{L}\sin\frac{n\pi x}{L}\sin\frac{n\pi ct}{L} + b_n\frac{n\pi c}{L}\sin\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}$$

$$\frac{\partial u}{\partial t}(x,0) = \sum_{n=1}^{\infty} b_n \frac{n\pi c}{L}\sin\frac{n\pi x}{L} = 0$$

Thus $b_n = 0$

thus $u(x,t) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L}\cos\frac{n\pi ct}{L}$

Determine a_n!

$\cancel{u(x,0) = \sum a_n \sin n\pi}$

$$u(x,0) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L} = \frac{1}{2}\sin\frac{\pi x}{L} + \frac{1}{4}\sin\frac{3\pi x}{L}$$

$$\Rightarrow a_1 \sin\frac{\pi x}{L} + a_2 \sin\frac{2\pi x}{L} + a_3 \sin\frac{3\pi x}{L} + \dots = \frac{1}{2}\sin\frac{\pi x}{L} + \frac{1}{4}\sin\frac{3\pi x}{L}$$

$$\left.\begin{aligned} &\Rightarrow a_1 = \tfrac{1}{2} \\ &a_2 = 0 \\ &a_3 = \tfrac{1}{4} \\ &a_4 = 0 \\ &\vdots \end{aligned}\right\} \Rightarrow u(x,t) = \frac{1}{2}\sin\frac{\pi x}{L}\cos\frac{\pi ct}{L} + \frac{1}{4}\sin\frac{3\pi x}{L}\cos\frac{3\pi ct}{L}$$