2.4 The Product and Quotient Rules

I Product Rule

Let $f(x) = u(x)\,v(x)$. Then

$$f'(x) = u'v + vu'$$

For proof see text.

Examples: Differentiate:

a) $y = (4x + 3x^2)(6 - 3x)$

b) $f(x) = \left(\frac{1}{x} + 1\right)(2x + 1)$

If you have 3 functions multiplied:

$$f(x) = u(x)\,v(x)\,w(x)$$

$$f'(x) = u'vw + uv'w + uvw'$$

The Quotient Rule

Let $f(x) = \frac{u(x)}{v(x)}$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

Examples: Differentiate

a) $y = \frac{x+4}{5x+3}$

b) $y = \frac{3-(2/x)}{x+4}$

c) $y = \frac{(1+x)(2x-1)}{x-1}$

2.5 The Chain Rule

The Chain Rule

If $y = f(u)$ is a differentiable function of u,
and $u = g(x)$ is a differentiable function of x,
then $y = f(g(x))$ is a differentiable function of x
and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or equivalently:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$$

Ex. Differentiate:

$$y = (x^3+1)^2$$

The General Power Rule

If $y = [u(x)]^n$, where u is a differentiable function of x and n is a real number, then

$$\frac{dy}{dx} = n[u(x)]^{n-1}\frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[u^n] = nu^{n-1}u'$$

Differentiate!

a) $y = (x^2+3x)^4$

b) $y = \sqrt[3]{(x+1)^2}$

c) $y = \frac{3}{x^2+1}$; $y = \frac{3}{(x+1)^2}$

d) $y = x^2\sqrt{x^2+1}$

e) $f(x) = \left(\frac{x+1}{x-5}\right)^2$