2.1: 9, 10a, b, 11

2.2: 5, 6, 7, 9, 11, 12

$\boxed{7a}$

$$g(x) = \begin{cases} \pi + x & -\pi \le x \le 0 \\ \pi - x & 0 \le x \le \pi \end{cases}$$

$$g(-x) = \begin{cases} \pi - x & -\pi \le -x \le 0 \\ \pi + x & 0 \le -x \le \pi \end{cases} =$$

$$= \begin{cases} \pi - x & 0 \le x \le \pi \\ \pi + x & -\pi \le x \le 0 \end{cases} = g(x)$$

$\Rightarrow$ Even! $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} (\pi + x)\,dx + \int_{0}^{\pi} (\pi - x)\,dx\right]$$

$$= \frac{1}{2\pi}\left[\pi x + \frac{1}{2}x^2\Big|_{-\pi}^{0} + \pi x - \frac{1}{2}x^2\Big|_{0}^{\pi}\right] =$$

$$= \frac{1}{2\pi}\left[+\pi^2 - \cancel{\frac{\pi^2}{2}} + \cancel{\pi^2} - \cancel{\frac{1}{2}\pi^2}\right] = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} (\pi + x)\cos nx\,dx + \int_{0}^{\pi} (\pi - x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\cancelto{0}{\sin nx}\Big|_{-\pi}^{0} + \frac{1}{n^2}\cos nx + \frac{x}{n}\cancelto{0}{\sin nx}\Big|_{-\pi}^{0}\right.$$

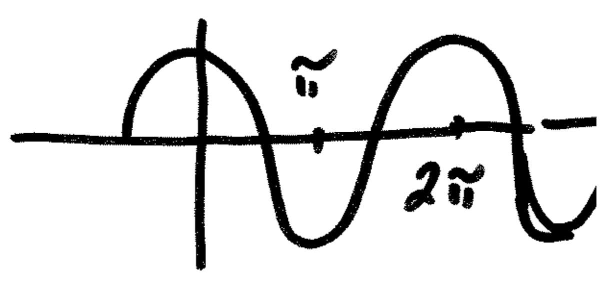
$$\left. + \frac{\pi}{n}\cancelto{0}{\sin nx}\Big|_{0}^{\pi} - \frac{1}{n^2}\cos nx - \frac{x}{n}\cancel{\sin nx}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{1}{n^2}\cos -(n\pi) - \frac{1}{n^2}\cos n\pi + \frac{1}{n^2}\right]$$

$\cos(-\alpha) = \cos\alpha$

$$= \frac{1}{\pi}\left[\frac{2}{n^2} - \frac{2}{n^2}\cos n\pi\right]$$

$$= \frac{2}{\pi}\frac{1}{n^2}\left[1 - \cos n\pi\right]$$



$$= \begin{cases} \frac{4}{\pi}\frac{1}{n^2} & \text{when } n \text{ is odd} \\ 0 & \text{when } n \text{ is even} \end{cases}$$

$$\Rightarrow g(x) = \frac{\pi}{2} + \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^2}\cos(2k+1)x$$

7b

$$h(x) = \begin{cases} \pi - x & 0 < x \leq \pi \\ 0 & \pi < x < 2\pi \end{cases}$$

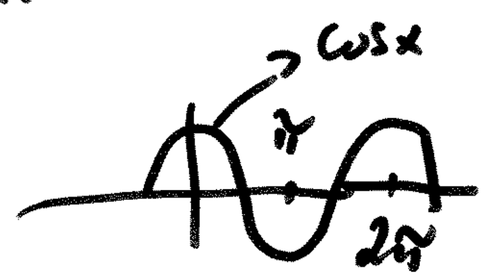
$$a_0 = \frac{1}{2\pi}\int_0^{\pi} (\pi - x)\,dx = \frac{1}{2\pi}\left[\pi x - \frac{1}{2}x^2\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\pi^2 - \frac{1}{2}\pi^2\right] = \frac{1}{4}\pi$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} (\pi - x)\cos nx\,dx =$$

$$= \frac{1}{\pi}\left[\cancelto{0}{\frac{\pi}{n}\sin nx}\Big|_0^{\pi} - \frac{1}{n^2}\cos nx - \cancelto{0}{\frac{x}{n}\sin nx}\Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\frac{1}{n^2}\cos n\pi + \frac{1}{n^2}\right] = \frac{1}{\pi}\frac{1}{n^2}\left[1 - \cos n\pi\right]$$



$$= \begin{cases} \frac{2}{\pi}\frac{1}{n^2} & n\text{-odd} \\ 0 & n\text{-even} \end{cases}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} (\pi - x)\sin nx \, dx =$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{n}\cos nx \Big|_0^{\pi} - \frac{1}{n^2}\cancelto{0}{\sin nx} + \frac{x}{n}\cos nx \Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{n}\cancel{\cos n\pi} + \frac{\pi}{n} + \frac{\pi}{n}\cancel{\cos n\pi}\right] = \frac{1}{n}$$

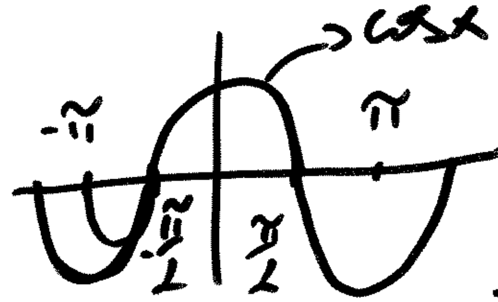
n

$$h(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi}\frac{1}{n^2}\left[1-(-1)^n\right]\cos nx + \frac{1}{n}\sin nx\right]$$

7c] $f(x) = |\cos x|$ if $-\pi \le x \le \pi$

$f(-x) = |\cos(-x)| = |\cos x| \Rightarrow f(x)$ - even

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |\cos x|\, dx =$$



$$= \frac{1}{2\pi}\left[-\int_{-\pi}^{-\frac{\pi}{2}} \cos x\, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx - \int_{\frac{\pi}{2}}^{\pi} \cos x\, dx \right]$$

$$= \frac{1}{2\pi}\left[-\sin x \Big|_{-\pi}^{-\frac{\pi}{2}} + \sin x \Big|_{-\frac{\pi}{2}}^{\pi/2} - \sin x \Big|_{\frac{\pi}{2}}^{\pi} \right]$$

$$= \frac{1}{2\pi}\left[-\left(\sin -\frac{\pi}{2} - \sin(-\pi)\right) + \sin\frac{\pi}{2} - \sin -\frac{\pi}{2} - \left(\sin\pi - \sin\frac{\pi}{2}\right) \right]$$

$$= \frac{1}{2\pi}\left[1 + 1 + 1 + 1 \right] = \frac{2}{\pi}$$

~~[illegible]~~

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x| \cos nx \, dx$$

$$= \frac{1}{\pi}\left[-\int_{-\pi}^{-\frac{\pi}{2}} \cos x \cos nx \, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \cos nx \, dx - \int_{\frac{\pi}{2}}^{\pi} \cos x \cos nx \, dx \right]$$

~~[illegible]~~

$$= \frac{1}{\pi}\left[- \frac{\sin[1-n]x}{2(1-n)} - \frac{\sin[1+n]x}{2(n+1)} \Bigg|_{-\pi}^{-\frac{\pi}{2}} \right.$$

$$+ \frac{\sin(1-n)x}{2(1-n)} + \frac{\sin(n+1)x}{2(n+1)} \Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

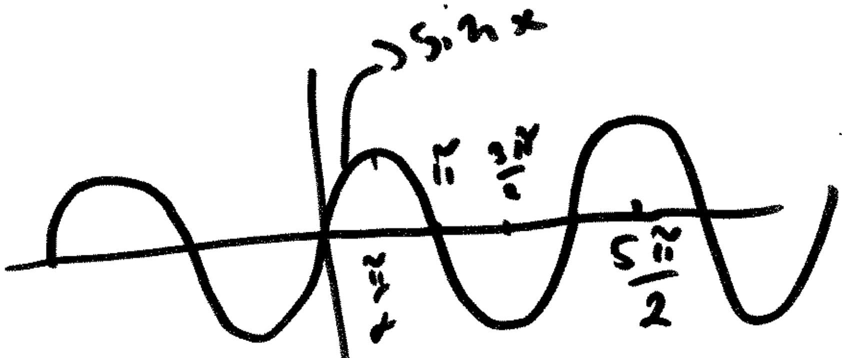
$$\left. - \frac{\sin(1-n)x}{2(1-n)} - \frac{\sin(1+n)x}{2(n+1)} \Bigg|_{\frac{\pi}{2}}^{\pi} \right]$$

$$= \frac{1}{\pi}\left[+ \frac{\sin(n-1)\frac{\pi}{2}}{2(n-1)} + \frac{\sin(n+1)\frac{\pi}{2}}{2(n+1)} - \frac{\sin(n-1)\pi}{2(n-1)} - \frac{\sin(n+1)\pi}{2(n+1)} \right.$$

$$+ \frac{\sin(n-1)\frac{\pi}{2}}{[illegible]} + \frac{\sin(n+1)\frac{\pi}{2}}{[illegible]} + \frac{\sin(n-1)\frac{\pi}{2}}{[illegible]} + \frac{\sin(n+1)\frac{\pi}{2}}{2(n+1)}$$

$$\frac{\sin(n-1)\pi}{2(n-1)} - \frac{\sin(n+1)\pi}{2(n+1)} + \frac{\sin(n-1)\frac{\pi}{2}}{2(n-1)} + \frac{\sin(n+1)\frac{\pi}{2}}{2(n+1)}$$

$$\frac{2\sin(n-1)\frac{\pi}{2}}{(n-1)} + \frac{2\sin(n+1)\frac{\pi}{2}}{(n+1)} - \cancel{\frac{\sin(n-1)\pi}{n-1}}^{\to 0} - \cancel{\frac{\sin(n+1)}{n+1}}$$



$$\frac{2}{\pi}\begin{cases} \frac{1}{n-1} - \frac{1}{n+1} & \text{when } n \text{ is even } n=2k \text{ and } k\text{-odd} \\ -\frac{1}{n-1} + \frac{1}{n+1} & n=2k,\ k\text{-even} \\ 0 & n\text{-odd} \end{cases}$$

$$\Rightarrow \frac{2}{\pi}\begin{cases} \frac{n+1-n+1}{(n-1)(n+1)} = \frac{2}{n^2-1} & n=2k,\ k\text{-odd} \\ -\frac{2}{n^2-1} & n=2k,\ k\text{-even} \\ 0 & n\text{-odd} \end{cases}$$

$2 \quad 4 \quad \cos(2k)x$

$b_n = 0$ for every n since $f(x) = |\cos x|$ is even

$$\Rightarrow f(x) = \frac{2}{\pi} - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)^2 - 1} \cos(2kx)$$

d) $f(x) = e^{-|x|}$ if $-\pi < x \leq \pi$

$f(-x) = e^{-|-x|} = e^{-|x|} \to$ even

$\Rightarrow b_n = 0$ for every n

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-|x|}\,dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0} e^{x}\,dx + \int_{0}^{\pi} e^{-x}\,dx\right]$$

$$= \frac{1}{2\pi}\left[e^{x}\Big|_{-\pi}^{0} - e^{-x}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[1 - e^{-\pi} - (e^{-\pi} - 1)\right] = \frac{1}{\pi}\left[1 - e^{-\pi}\right]$$

$$= \frac{e^{\pi} - 1}{\pi e^{\pi}}$$

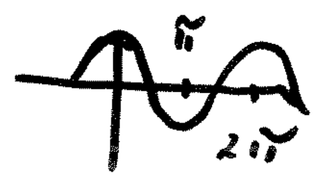
$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} e^{x}\cos nx\,dx + \int_{0}^{\pi} e^{-x}\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{x}}{1+n^2}\left(\cos nx + \cancelto{0}{n\sin nx}\right)\Bigg|_{-\pi}^{0}\right.$$

$$\left. + \frac{e^{-x}}{1+n^2}\left(-\cos nx + \cancelto{0}{n\sin nx}\right)\Bigg|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{1+n^2} - \frac{e^{-\pi}}{1+n^2}\cos(-n\pi) + \frac{e^{-\pi}}{1+n^2}(-\cos n\pi)\right.$$

$$\left. + \frac{1}{1+n^2}\right]$$



$$= \frac{1}{\pi}\left[\left[\frac{2}{n^2+1}\right] - \frac{2e^{-\pi}}{n^2+1}\cos n\pi\right]$$

$$= \frac{\frac{2}{\pi e^{\pi}}}{n^2+1}\left[\begin{array}{ll} e^{\pi} - 1 & \text{when } n \text{ is even} \\ e^{\pi} + 1 & \text{when } n \text{ is odd} \end{array}\right.$$

$$\Rightarrow f(x) = \frac{e^{\pi}-1}{\pi e^{\pi}} + \frac{2}{\pi e^{\pi}}\sum_{n=1}^{\infty}\frac{1}{n^2+1}\left[e^{\pi} - (-1)^n\right]\cos nx$$