HW 5 key

3.5		5.1
1 –	5pts	1 – 5
2	10	2 – 10
9	5	12 – 5
11	10	13 – 5

/55

.5

2

$$b_n = \frac{2}{\pi}\int_0^{\pi} 30 \sin x \sin nx \, dx = \begin{cases} 30 & \text{if } n = 1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

$$\lambda_n = n \implies \lambda_1 = 1$$

$$u(x,t) = 30 e^{-t} \sin x$$

#11

$$u_1(x) = \frac{T_2 - T_1}{L} x + T_1 = \frac{0-100}{1} x + 100 = \underline{-100x + 100}$$

$$B_n = 2\int_0^1 (30 \sin \pi x + 100x - 100) \sin n\pi x \, dx$$

$$= 60\int_0^1 \sin \pi x \sin n\pi x \, dx + 200\int_0^1 x \sin n\pi x \, dx - 200 \int_0^1 \sin n\pi x \, dx$$

$$= \begin{cases} 30 \\ + \\ 0 \end{cases} + \frac{200}{n^2\pi^2} \sin n\pi x \Big|_0^1 - \frac{200x}{n\pi} \cos n\pi x \Big|_0^1 + \frac{200}{n\pi} \cos n\pi x \Big|_0^1$$

$$= \begin{cases} \cancel{30 - \frac{200}{\pi}\cos\pi} + \cancel{\frac{200}{\pi}} = \boxed{30 + \frac{200}{\pi}} & n = 1 \\ -\cancel{\frac{200}{n\pi}}(-1)^n \cancel{+ 200} + \cancel{\frac{200}{n\pi}}(-1)^n \cancel{\ldots} + \boxed{\frac{200}{n\pi}} & n > 1 \end{cases}$$

$$u(x,t) = -100(x-1) + \left(30 + \frac{200}{\pi}\right) \sin \pi x \, e^{-\pi^2 t}$$

$$+ \frac{200}{\pi} \sum_{n=2}^{\infty} \frac{1}{n} e^{-(n\pi)^2 t} \sin n\pi x$$

or

$$u(x,t) = -100x + 100 + 30 \sin(\pi x) e^{-\pi^2 t} + \sum_{n=1}^{\infty} \frac{200}{\pi} \sin(\pi n x) e^{-\pi^2 n^2 t}$$

3.7

\# 2

$$B_{mn} = 4\int_0^1\int_0^1 \sin\pi x \sin\pi y \sin m\pi x \sin n\pi y \, dx\, dy$$

By orthogonality $= \begin{cases} 1 \text{ if } m=n=1 \\ 0 \text{ otherwise} \end{cases}$

$$\Rightarrow B_{11} = 1 \qquad \lambda_{11} = \sqrt{1+1} = \sqrt{2}$$

$$B^*_{mn} = \frac{4}{\lambda_{mn}} \int_0^1\int_0^1 \sin\pi x \sin m\pi x \sin n\pi y \, dx\, dy$$

$$= \begin{cases} \frac{2}{\lambda_{1n}} \int_0^1 \sin n\pi y \, dy & \text{if } m=1 \\ 0 & \text{if } m\neq 1 \end{cases}$$

$$\Rightarrow B^*_{1n} = -\frac{2}{n\pi\lambda_{1n}}(\cos n\pi - 1) =$$

$$= \begin{cases} \frac{4}{n\pi\lambda_{1n}} & n\text{-odd} \\ 0 & n\text{-even} \end{cases}$$

$$\Rightarrow u(x,y,t) = \cos\sqrt{2}\,t \sin\pi x \sin\pi y$$

$$+ \frac{4}{\pi}\sum_{k=0}^{\infty} \frac{\sin\sqrt{1+(2k+1)^2}\,t}{(2k+1)\sqrt{1+(2k+1)^2}} \sin\pi x \sin(2k+1)\pi y$$