Section 3.4:

#2	10
#4	5
#6	5
#13	10
#14	10

#2 $f(x) = \sin \pi x \cos \pi x$ $\qquad c = \frac{1}{\pi}$

$g(x) = 0$

$$u(x,t) = \frac{1}{2}\left[f^*(x+ct) + f^*(x-ct)\right]$$

$$f(x) = \sin \pi x \cos \pi x = \frac{1}{2} \sin 2\pi x$$

$$f^*(x) = \frac{1}{2} \sin 2\pi x$$

$$\underline{u(x,t) = \frac{1}{4}\left[\sin 2\pi(x+ct) + \sin 2\pi(x-ct)\right]}$$

This is fine as on answer

They may have gone 1 step further:

$$u(x,t) = \frac{1}{4}\left[\sin 2\pi x \cos 2\pi ct + \cancel{\cos 2\pi x \sin 2\pi ct} + \sin 2\pi x \cos 2\pi ct - \cancel{\cos 2\pi x \sin 2\pi ct}\right]$$

$$\underline{= \frac{1}{2} \sin 2\pi x \cos 2\pi ct}$$

13 I gave them hints how to prove this, so just grade this part:

They must know how to prove that

$$\frac{1}{2}\left(f^*\left(x+c\left(t+\frac{L}{c}\right)\right)+f^*\left(x-c\left(t+\frac{L}{c}\right)\right)\right)=-\frac{1}{2}\left(f^*(x+ct)+f^*(x-ct)\right)$$

$$f^*\left(x+c\left(t+\frac{L}{c}\right)\right)=f^*(x+ct+L)=f^*\left(L-(-x-ct)\right)$$

$$=f^*(-x-ct)=-f^*(x+ct)$$

since f^* is odd

$$f^*\left(x-c\left(t+\frac{L}{c}\right)\right)=f^*(x-ct-L)=$$

$$=-f^*(L-x+ct)=-f^*\left(L-(x-ct)\right)=-f^*(x-ct)$$

✓

#14 I gave ~~hints~~ how to prove part of the problem. Just grade this:

$$u(x,t) = \frac{1}{2}\left[f^*(x-ct) + f^*(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g^*(s)\,ds$$

$$u(x,0) = \frac{1}{2}\left[f^*(x) + f^*(x)\right] + \underbrace{\frac{1}{2c}\int_x^x g^*(s)\,ds}_{=0}$$

$$u(x,0) = f^*(x) \overset{?}{=} f(x) \quad \text{when } 0 < x < L$$

$$\frac{\partial u}{\partial t}(x,t) = \frac{1}{2}\left[f^{*\prime}(x-ct)(-c) + f^{*\prime}(x+ct)(c)\right] + \frac{1}{2c}\left[g^*(x+ct)(c) - g^*(x-ct)(-c)\right]$$

$$= \frac{c}{2}\left[-f^{*\prime}(x-ct) + f^{*\prime}(x+ct)\right] + \frac{1}{2}\left[g^*(x+ct) + g^*(x-ct)\right]$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{c}{2}\left[-f^{*\prime}(x) + f^{*\prime}(x)\right] + \frac{1}{2}\left[g^*(x) + g^*(x)\right]$$

$$= g^*(x) = g(x) \quad \text{when } 0 < x < L$$