HW Set #2

2.1

\# 9 - 5 pts
\# 10 5 pts
\# 11 - 5 pts

15 pts

2.2

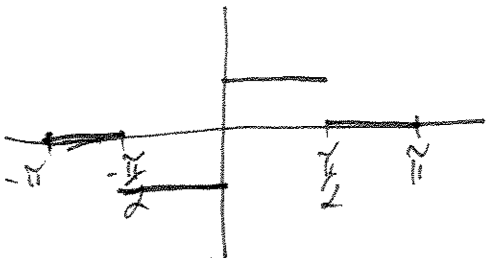
\# 5 - 5 pts
\# 6 - 10 pts
\# 7 - 10 pts
\# 9 - 5 pts
\# 11 - 10 pts
\# 12 - 5 pts

45 pts

Total Points: 60 pts

2.2

#6 $f(x) = \begin{cases} 1 & \text{if } 0 < x < \pi/2 \\ -1 & \text{if } -\pi/2 < x < 0 \\ 0 & \text{if } \frac{\pi}{2} < |x| < \pi \end{cases}$



$f(x)$ is odd $\Rightarrow$

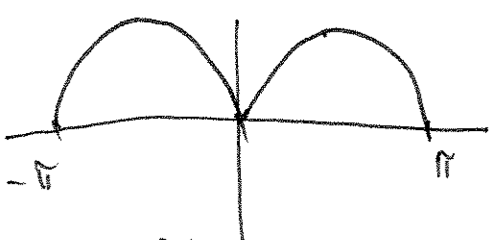
$\Rightarrow a_0 = 0 \quad a_n = 0$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cancel{\sin nx\,dx} = \frac{2}{\pi}\int_0^{\pi/2} \sin nx\, dx$$

$$= -\frac{2}{\pi}\,\frac{1}{n}\cos nx \Big|_0^{\pi/2} = -\frac{2}{\pi}\,\frac{1}{n}\left(\cos\frac{n\pi}{2} - 1\right)$$

$\Rightarrow$ Fourier series : $\frac{2}{\pi}\sum \frac{1}{n}\left(1 - \cos\frac{n\pi}{2}\right)\sin nx$

#7 $f(x) = |\sin x| \quad -\pi \le x \le \pi$



$f(x)$ - even $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} \sin x\, dx = -\frac{1}{\pi}\cos x\Big|_0^{\pi} = -\frac{1}{\pi}\left[-1-1\right] = \frac{2}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin x \cos nx \, dx$$

$$= \frac{2}{\pi}\left[\frac{\cos[(n-1)x]}{2(n-1)} - \frac{\cos[(n+1)x]}{2(n+1)}\right]\Bigg|_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\cos[(n-1)\pi]}{2(n-1)} - \frac{\cos[(n+1)\pi]}{2(n+1)} - \frac{1}{2(n-1)} + \frac{1}{2(n+1)}\right]$$

$$= \frac{2}{\pi}\begin{cases} \dfrac{1}{2(n-1)} - \dfrac{1}{2(n+1)} - \dfrac{1}{2(n-1)} + \dfrac{1}{2(n+1)} = 0 & \text{when } n\text{-odd or equivalently, } (n+1), (n-1)\text{-even} \\ -\dfrac{1}{2(n-1)} + \dfrac{1}{2(n+1)} - \dfrac{1}{2(n-1)} + \dfrac{1}{2(n+1)} & \text{when } n\text{-even} \end{cases}$$

$$= -\frac{1}{n-1} + \frac{1}{n+1} = \frac{-n-1+n-1}{n^2-1} = -\frac{2}{n^2-1}$$

$$= \begin{cases} 0 & n\text{-odd} \\ -\dfrac{4}{\pi}\dfrac{1}{n^2-1} & n\text{-even} \end{cases}$$

$\Rightarrow$ let $n = 2k$ to pick up only the even a_n

Fourier series: $\dfrac{2}{\pi} - \dfrac{4}{\pi}\displaystyle\sum_{k=0}^{\infty} \frac{1}{(2k)^2-1}\cos 2kx$

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$f(x) = \sin^2 x$ even $\Rightarrow$ $b_n = 0$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2 x \, dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1-\cos 2x}{2} \, dx$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} (1-\cos 2x)\, dx = \frac{1}{4\pi}\left[x - \frac{1}{2}\sin 2x \right]_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi}\left[\pi - 0 + \pi + 0 \right] = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin^2 x \cos nx \, dx = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} (1-\cos 2x)\cos nx \, dx \right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} \cos nx \, dx - \int_{-\pi}^{\pi} \cos 2x \cos nx \, dx \right]$$

$$= \frac{1}{2\pi}\left[\underbrace{\frac{1}{n}\sin nx \Big|_{-\pi}^{\pi}}_{0} - \underbrace{\int_{-\pi}^{\pi} \cos 2x \cos nx \, dx}_{= \begin{cases} \pi & \text{when } n=2 \\ 0 & \text{when } n \neq 2 \end{cases}} \right]$$

orthogonal of $\sin nx$ $\cos nx$

$$= \begin{cases} -\frac{1}{2} & \text{when } n = 2 \\ 0 & \text{when } n \neq 2 \end{cases}$$

$$\Rightarrow \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$f(x) = \cos^2 x = 1 - \sin^2 x =$$

$$= 1 - \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) = \frac{1}{2} + \frac{1}{2}\cos 2x$$

(by linearity of Fourier Series)