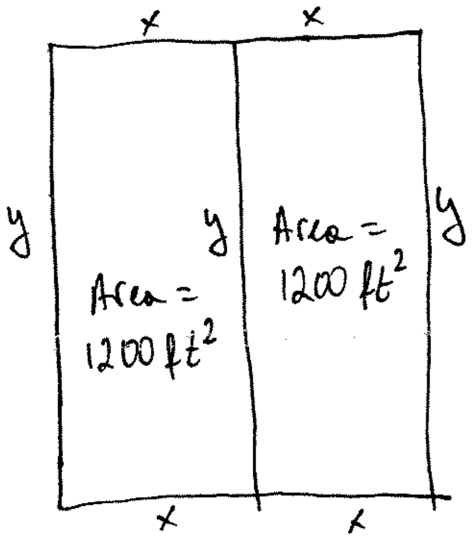
MATH1100 – QUIZ 6

NAME:

1. (10 points) Two equal rectangular lots are enclosed by fencing the perimeter of a rectangular lot and then putting a fence across its middle. If each lot is to contain 1200 square feet, what is the minimum amount of fence needed to enclose the lots (include the fence across the middle)?



$P = 4x + 3y$

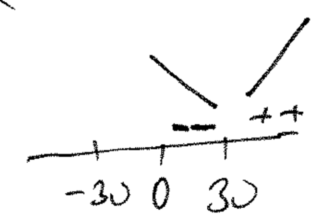
$xy = 1200 \Rightarrow y = \frac{1200}{x}$

$P = 4x + 3\,\frac{1200}{x} = 4x + \frac{3600}{x}$

$P'(x) = 4 - \frac{3600}{x^2}$

$4 - \frac{3600}{x^2} = 0 \Rightarrow x^2 = 400$

$x = \pm 30$



$\Rightarrow x = 30$ ft is a min

$\Rightarrow y = \frac{1200}{30} = 40$ ft

2. (10 points) For the function $f(x) = \frac{x^2+4}{x}$ do the following:

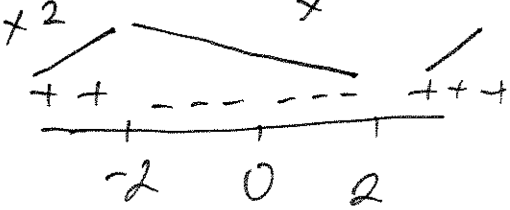
- Find the domain;
- Find horizontal and vertical asymptotes (if any);
- Find the first derivative and any relative min/max/horizontal points of inflection (if any);
- Find second derivative and the concavity of the graph of the function;
- Use this information (and additional points if you need to) to sketch the graph.

Domain: $x \neq 0$

HA: none ; VA $x = 0 \Rightarrow$ the y-axis is the V. A.

$f'(x) = \frac{x(2x) - (x^2+4)(1)}{x^2} = \frac{2x^2 - x^2 - 4}{x^2} = \frac{x^2-4}{x^2} = \frac{(x-2)(x+2)}{x^2}$

$\Rightarrow$ crit. values $x = \pm 2$, $x = 0$

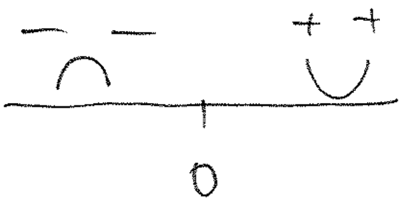


$\Rightarrow x = -2$ rel. max

$x = 0 \to$ asymptote

$x = 2$ rel. min

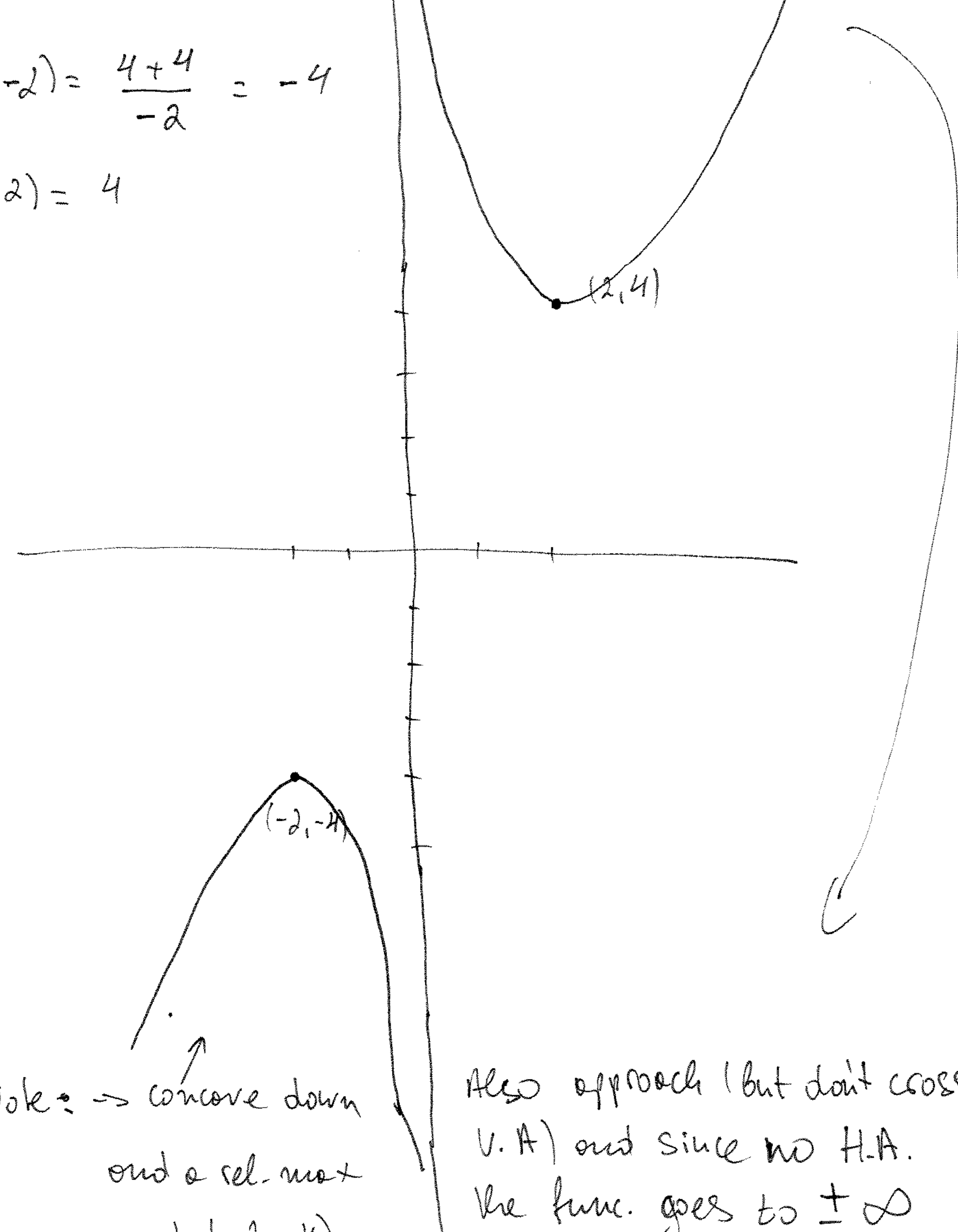
$f''(x) = \frac{x^2(2x) - (x^2-4)2x}{x^4} = \frac{2x^3 - 2x^3 + 8x}{x^4} = \frac{8x}{x^4} = \frac{8}{x^3}$

⇒ 

$f(-2) = \dfrac{4+4}{-2} = -4$

$f(2) = 4$

Note: concave up and rel. min at (2,4)



Note: → concave down and a rel. max at (-2,-4)

Also approach (but don't cross V.A) and since no H.A. the func. goes to $\pm\infty$