

MATH1100 - QUIZ 3 Solutions

NAME:

Determine whether the function is continuous on the whole real line, and if not find where the discontinuities occur:

1. (5 points)

$$f(x) = \frac{x+2}{x^2-4}$$

The discontinuities occur when the denominator is 0.

$$x^2 - 4 = (x-2)(x+2)$$

$$(x-2)(x+2) = 0 \Rightarrow \text{Discontinuous at}$$

continuous in the intervals $(-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$ $x = \pm 2$

2. (5 points)

$$f(x) = \frac{1}{x^2+4}$$

Check the denominator $x^2+4=0 \Rightarrow x^2=-4$ The equation does not have a solution, so no x can make the denominator 0.

Thus, the function is continuous everywhere $(-\infty, \infty)$

3. (5 points) Find the derivative $f'(x)$ of the function $f(x) = x^2 - 4$

using the definition $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - 4 - (x^2 - 4)}{\Delta x} =$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 4 - x^2 + 4}{\Delta x} =$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} =$$

$$= \lim_{\Delta x \rightarrow 0} 2x + \Delta x = 2x$$