Midterm 3 Review Questions

I

Tumor growth **For Problems 23 and 24, suppose that a tumor in a person's body has a spherical shape and that treatment is causing the radius of the tumor to decrease at a rate of 1 millimeter per month.**

23. At what rate is the volume decreasing when the radius is 3 millimeters? (Recall that $V = \frac{4}{3}\pi r^3$.)
24. At what rate is the surface area of the tumor decreasing when the radius is 3 mm? (Recall that for a sphere, $S = 4\pi r^2$.)

II

Cell growth A bacterial cell has a spherical shape. If the volume of the cell is increasing at a rate of 4 cubic micrometers per day, at what rate is the radius of the cell increasing when it is 2 micrometers? (Recall that for a sphere, $V = \frac{4}{3}\pi r^3$.)

III

Water purification Assume that water is being purified by causing it to flow through a conical filter that has a height of 15 inches and a radius of 5 inches. If the depth of the water is decreasing at a rate of 1 inch per minute when the depth is 6 inches, at what rate is the volume of water flowing out of the filter at this instant?

IV

Ladder safety A 30-ft ladder is leaning against a wall. If the bottom is pulled away from the wall at a rate of 1 ft/sec, at what rate is the top of the ladder sliding down the wall when the bottom is 18 ft from the wall?

V

Elasticity The demand function for a product is given by

$$p^3 + q^3 = 9.$$

(a) Find the price elasticity of demand when $x = 2$.

(b) Find the values of q and p that maximize the total revenue.

(c) For the value of q found in part (b), show that the price elasticity of demand has unit elasticity.

VI

Elasticity The demand function for a product is given by

$$p = 20 - 0.02q, \quad 0 < q < 1000.$$

(a) Find the price elasticity of demand when $x = 560$.

(b) Find the values of q and p that maximize the total revenue.

(c) For the value of q found in part (b), show that the price elasticity of demand has unit elasticity.

VII

VII Suppose that the demand for a product is given by

$$(p+1)\sqrt{q+1} = 1000$$

a) Find elasticity of demand when $p = \$39$

b) What type of elasticity is it?

c) How would a price increase affect the revenue

VIII If the demand and supply functions for a product are $p = 2100 - 3q$ and $p = 300 + 1.5q$, respectively, find the tax per unit t that will maximize the tax revenue T.

IX If the monthly demand function is $p = 7230 - 5q^2$ and the supply function before taxation is $p = 30 + 30q^2$, what tax per unit will maximize the tax revenue T.

X If the demand and supply functions for a product are $p = 5000 - 20q - 0.7q^2$ and $p = 500 + 10q + 0.3q^2$, respectively, find the tax per unit t that will maximize the tax revenue T.

Integrate:

a) $\int \left(-\frac{9}{x^4}\right) dx$

b) $\int \left(4x^3 - \frac{1}{x^2}\right) dx$

c) $\int \left(1 - \frac{1}{\sqrt[3]{x^2}}\right) dx =$

d) $\int 3x^2 \sqrt{x^3+1}\, dx$

e) $\int x(1-2x^2)^3 dx$

f) $\int \frac{x+1}{(x^2+2x-3)^2} dx$

g) $\int \frac{x-2}{\sqrt{x^2-4x+3}} dx$

h) $\int \frac{4y}{\sqrt{1+y^2}} dy$

i) $\int \frac{t+2t^2}{\sqrt{t}} dt$

j) $\int 9xe^{-x^2} dx$

k) $\int (2x+1)e^{x^2+x} dx$

l) $\int 3(x-4)e^{x^2-8x} dx$

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f) $\int \frac{x}{x^2+4} dx$

g) $\int \frac{x^2}{3-x^3} dx$

h) $\int \frac{x+3}{x^2+6x+7} dx$

i) $\int \frac{x^3-4x^2+3}{x-3} dx$

XII If the monthly marginal cost for a product is $\overline{MC} = 2x + 100$, with fixed costs amounting to \$200, find the total cost function for the month.

XIII A certain firm's marginal cost for a product is $\overline{MC} = 6x + 60$, its marginal revenue is $\overline{MR} = 180 - 2x$, and its total cost of production of 10 items is \$1000.

a) Find the optimal level of production.

b) Find the profit function

c) Find the profit or loss at the optimal level of production

XIV If consumption is \$8 bill when income is 0, and if the marginal propensity to consume is

$$\frac{dC}{dy} = 0.3 + \frac{0.2}{\sqrt{y}} \quad \text{(in bill. of dollars)}.$$

find the national consumption function.

RW

XV If consumption is \$5.8 bill when disposable income is 0, and if the marginal propensity to consume is

$$\frac{dC}{dy} = \frac{1}{\sqrt{2y+9}} + 0.8 \quad \text{(in bill of \$)}$$

find the national consumption function.

~~XV~~ XVI Find the particular solution of the diff equation:

a) $\frac{dy}{dx} = \frac{x^2}{y^4}$ when $x=1$; $y=1$

b) $\frac{dy}{dx} = \frac{x+1}{xy}$ when $x=1$, $y=3$

c) $\frac{dy}{dx} = \frac{(x^3+1)}{x^2 e^{2y}}$ when $x=1$, $y=0$

d) $\frac{dy}{dx} = \frac{1}{xy}$ when $x=1$, $y=3$

e) ~~$\frac{dy}{dx}$~~ $xe^y dx = (x+1) dy$ when $x=0$, $y=0$

f) $2xy\frac{dy}{dx} = y^2+1$ when $x=1$, $y=2$