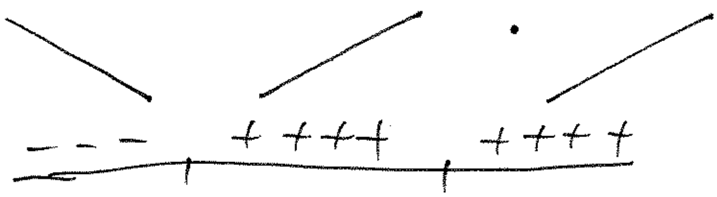
Test #2 Review Solutions

I a) $y' = x^3 - 2x^2 + x = x(x^2 - 2x + 1)$

$= x(x-1)^2$ $\quad$ $y' = 0 \Rightarrow x = 0 \quad x = 1$

are critical values



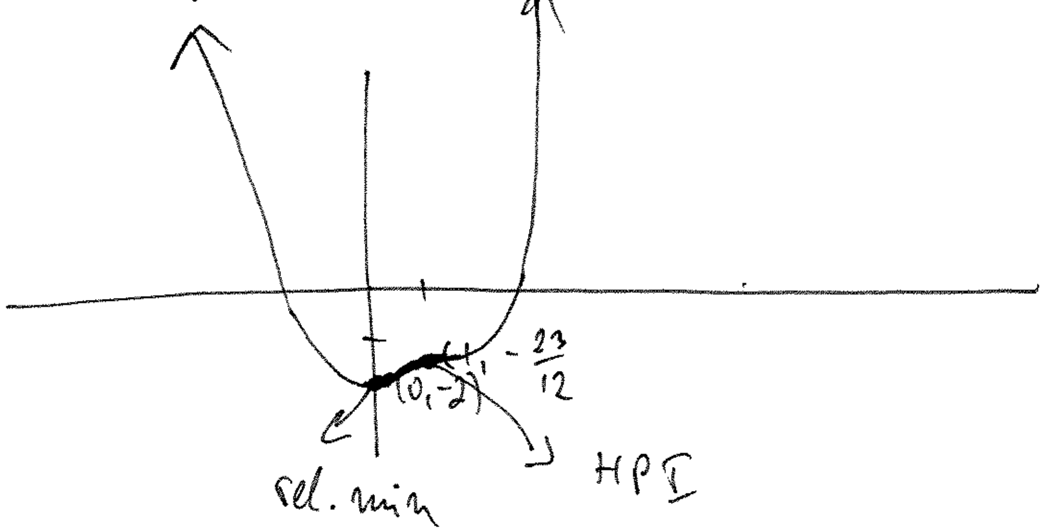
0 → rel. min

1 → HPI (horizontal point of inflection)

$f(0) = -2$

$f(1) = \underbrace{\frac{1}{4} - \frac{2}{3} + \frac{1}{2}}_{12} - 2 = \frac{3}{12} - \frac{8}{12} + \frac{6}{12} - \frac{24}{12}$

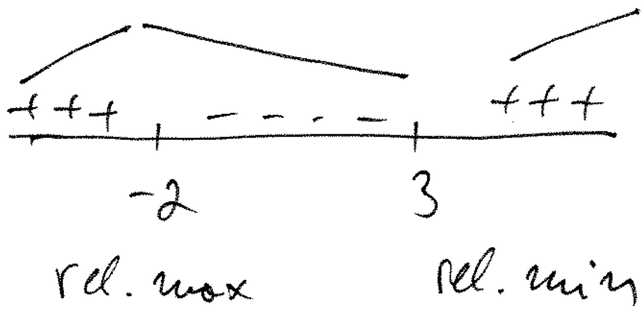
$= -\frac{23}{12} = -2\frac{11}{12}$



I 2) $f'(x) = 3x^2 - 3x - 18 =$

$= 3(x^2 - x - 6) = 3(x-3)(x+2)$

$\Rightarrow$ critical values $x = 3$ $x = -2$

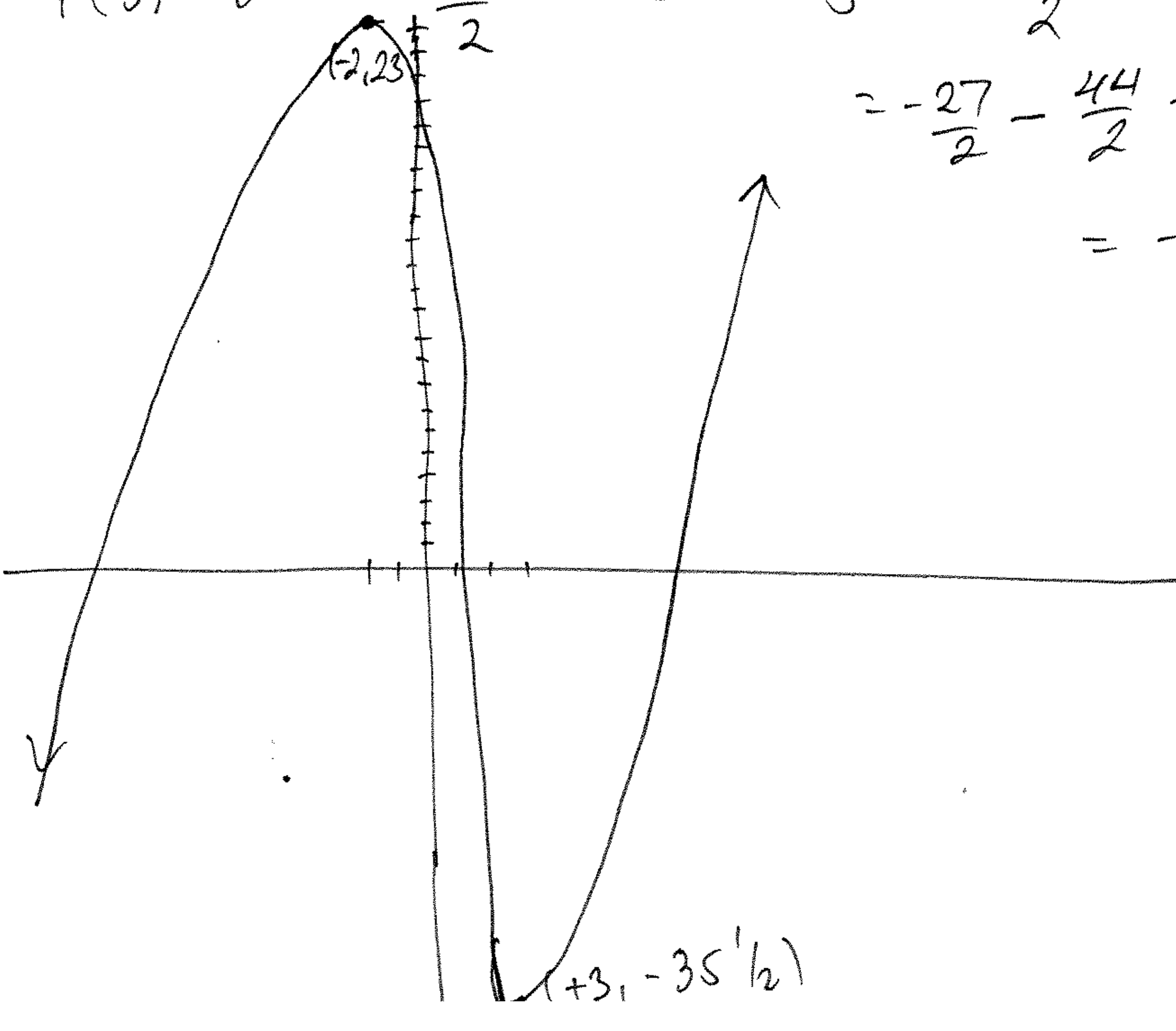


$f(-2) = -8 - 6 + 32 + 5 = 23$

$f(3) = 27 - \frac{27}{2} - 54 + 5 = -\frac{27}{2} - 22 =$

$= -\frac{27}{2} - \frac{44}{2} = \frac{-71}{2}$

$= -35\frac{1}{2}$



#3: $y' = x^5 - 4x^3 = x^3(x^2-4) =$

$= x^3(x-2)(x+2)$ → critical values

$x=0 \quad x=\pm 2$

$---\quad ++++\quad ---\quad ++++$

$-2 \qquad 0 \qquad 2$

-2 → rel. min

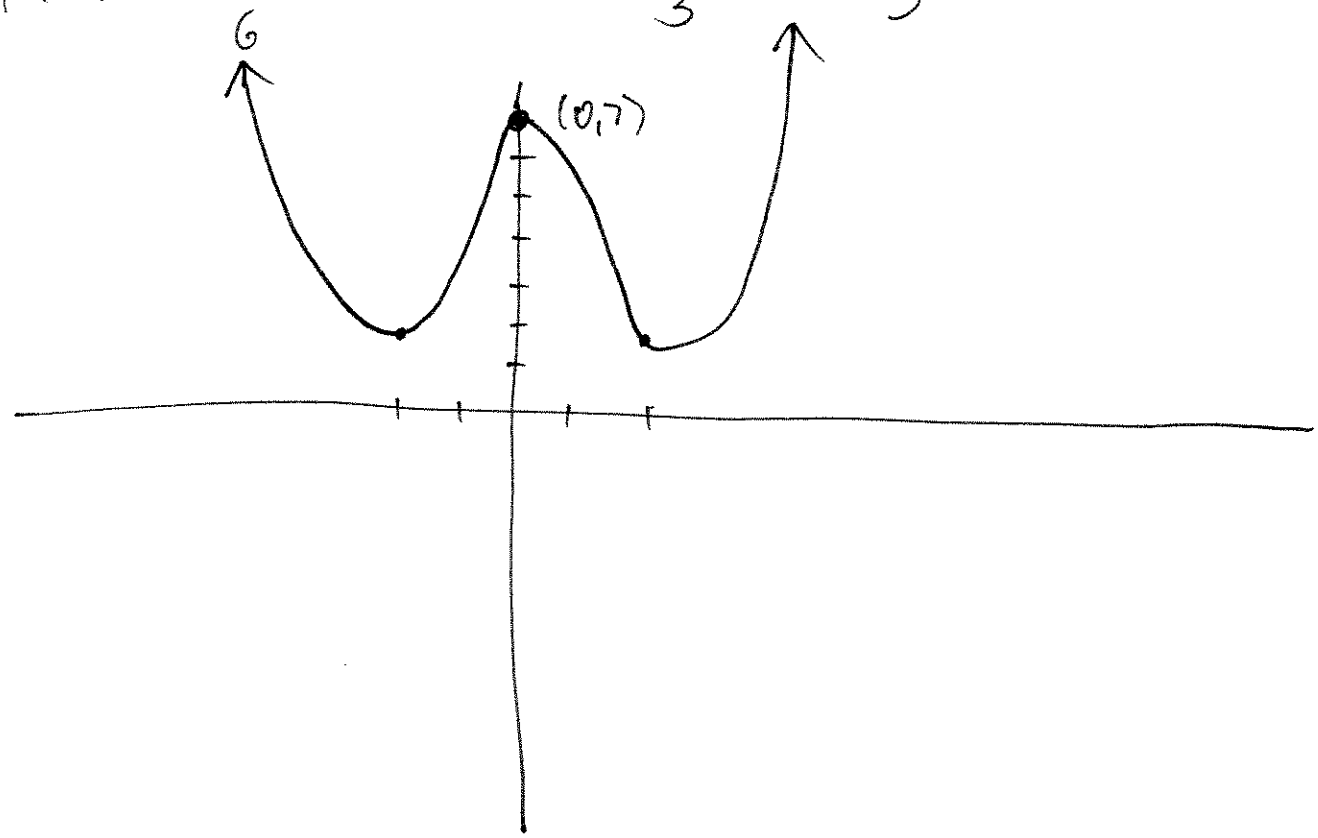
0 → rel. max

2 → rel. min

$$f(-2) = \frac{64}{6} - 16 + 7 = \frac{32}{3} - 9 = \frac{32-27}{3} = \frac{5}{3}$$

$$f(0) = 7$$

$$f(2) = \frac{64}{6} - 16 + 7 = \frac{5}{3} = 1\frac{2}{3}$$



I 4) $g'(x) = -\frac{2}{3}(x-3)^{-1/3} = \dfrac{-2}{3\sqrt[3]{x-3}}$

Remember critical values are particular x s where the 1^{st} derivative $=0$ <u>and</u> where the first derivative is undefined.

In this case there are no x s s.th. $g'(x)=0$

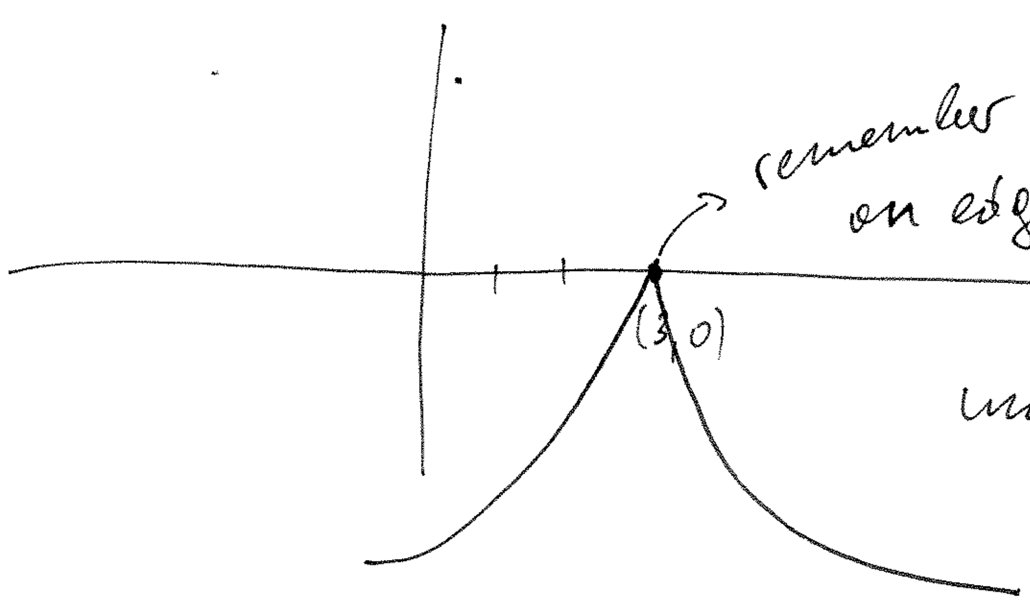
but when $x=3$ the denominator of $g'(x)$ is 0 and thus at $x=3$ $g'(x)$ is undefined

$g(3) = 0$

$+ + + + \;|\; - - - -$

$3 \Rightarrow$ HPI

$g(x) = -\sqrt[3]{(x-3)^2}$



remember you have an edge when $g'(x)$ is undefined

I 5) $y' = \frac{2}{3} x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

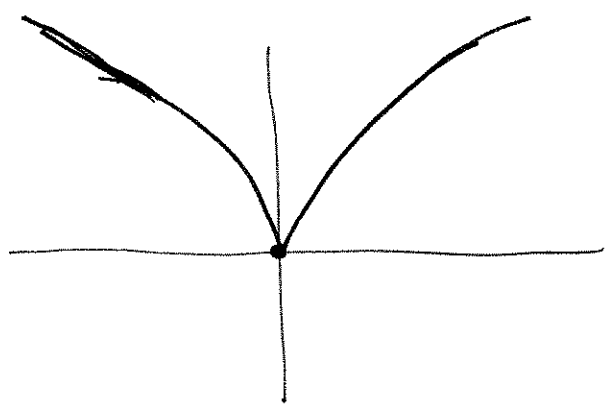
$\Rightarrow x = 0$ is a critical value (since it makes y' undefined)

$- - - -$ $\quad$ $+ + +$

$0 \Rightarrow$ rel. min

$y = \sqrt[3]{x^2}$

$f(0) = 0$



II 1) $y' = 3x^2 - 2x$

$y'' = 6x - 2$ $\qquad$ $6x - 2 = 0 \Rightarrow x = \frac{1}{3}$

$- -$ $\quad$ $+ +$

$\cap$ $\quad$ $\cup$

$\frac{1}{3} \Rightarrow$ point of inflection

2) $y' = 3x^2 - 6x$

$y'' = 6x - 6$ $\qquad$ $6x - 6 = 0 \Rightarrow x = 1$

$- -$ $\quad$ $+ +$

$\cap$ $\quad$ $\cup$

$1 \Rightarrow$ point of inflection

3) $y' = 4x^3 - 24x^2 + 32x$

$$y'' = 12x^2 - 48x + 32$$

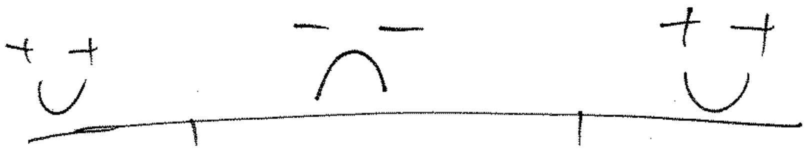
$$= 4(3x^2 - 12x + 8)$$

$$4(3x^2 - 12x + 8) = 0$$

$$\Rightarrow 3x^2 - 12x + 8 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{144 - 4(3)(8)}}{6} =$$

$$= \frac{12 \pm \sqrt{48}}{6} = \frac{12 \pm \sqrt{16 \times 3}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = 2 \pm \frac{2}{3}\sqrt{3}$$



$2 - \frac{2}{3}\sqrt{3}$ ≈ 0.845

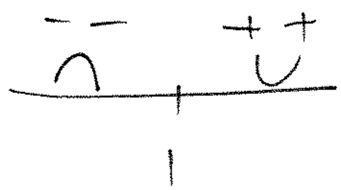
$2 + \frac{2}{3}\sqrt{3}$ ≈ 3.155

points of inflection

II 4) $y' = x^2 - 2x - 3$

$y'' = 2x - 2$ $\qquad 2x - 2 = 0$

$\qquad\qquad\qquad x = 1$



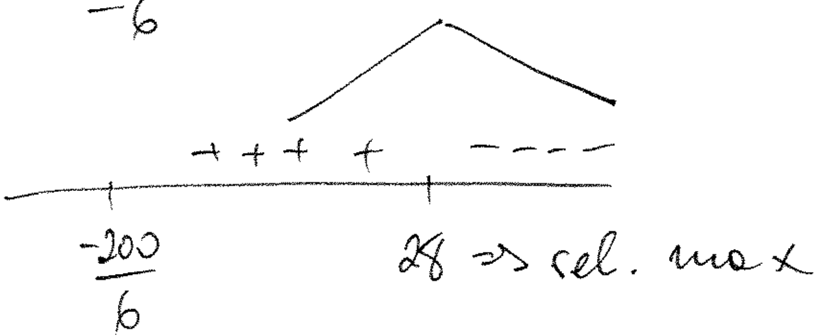
III $R'(x) = 2800 - 16x - 3x^2$

$2800 - 16x - 3x^2 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{16 \pm \sqrt{256 - 4(-3)(2800)}}{-6}$$

$$= \frac{16 \pm \sqrt{33856}}{-6} = \frac{16 \pm 184}{-6} = \begin{cases} \frac{-200}{6} \\ 28 \end{cases}$$

$\frac{-200}{6} \Rightarrow$ negative $\Rightarrow$ no



Max revenue $R(28) = 2800(28) - 8(28)^2 - (28)^3 =$

$= 78400 - 6272 - 21952 = 50{,}176$

IV a) $\bar{C} = \dfrac{C(x)}{x} = 0.001x^2 + 5 + \dfrac{250}{x}$

$\bar{C}' = 0.002x - \dfrac{250}{x^2}$

$0.002x - \dfrac{250}{x^2} = 0$ / multiply both sides by x^2

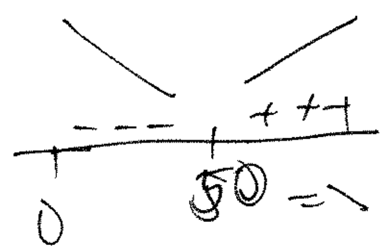
$0.002x^3 - 250 = 0$

$0.002x^3 = 250$

$x^3 = \dfrac{250}{0.002}$

$x^3 = 125000$

$x = 50$



$\Rightarrow$ rel. min

b) There is a mistake in the statement of the problem (You will not be able to solve for x easily (without a computer)).
Solve instead:

$C = x^3 + 12x^2 + 48x + 64$

$\bar{C} = x^2 + 12x + 48 + \dfrac{64}{x}$

$\bar{C}' = 2x + 12 - \dfrac{64}{x^2}$

$2x + 12 - \dfrac{64}{x^2} = 0 \rightarrow 2x^3 + 12x^2 - 64 = 0$

$x^3 + 6x^2 - 32 = 0$

$x = 2$ is a solution:

$2^3 + 6(2)^2 - 32 =$

$= 8 + 24 - 32 = 0$

IV b) $x^3+6x^2-32=0$

and $x=2$ is a solution

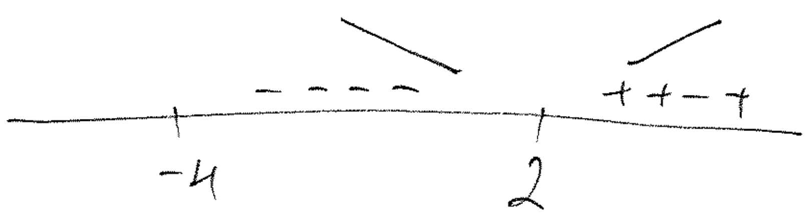
Divide it out:

$$
\begin{array}{r}
x^2+8x+16 \\
(x-2)\,\overline{\big)\,x^3+6x^2+0x+32} \\
-x^3+2x^2 \\
\hline
8x^2+0x \\
-8x^2+16x \\
\hline
16x+32 \\
-16x+32 \\
\hline
0
\end{array}
$$

$\Rightarrow (x-2)(x^2+8x+16)=0$

$(x-2)(x+4)^2=0$

$\Rightarrow x=2 \quad x=-4$ are critical values



$\Rightarrow x=2$ will minimize the average cost

(This problem is a bit too hard for the test)

V $\quad R(x) = \text{price} \times \text{quantity} = px = (100 - 0.5x^2)x$

$= 100x - 0.5x^3$

$P(x) = R(x) - C(x) = 100x - 0.5x^3 - 40x - 37.5$

$= -0.5x^3 + 60x - 37.5$

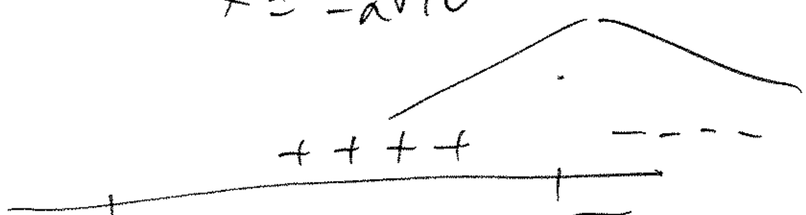
$P'(x) = -1.5x^2 + 60$

$-1.5x^2 + 60 = 0$

$-1.5x^2 = -60$

$x^2 = 40$

$x = \pm 2\sqrt{10}$



$-2\sqrt{10}$ $\qquad 2\sqrt{10} \rightarrow$ rel. max

price $= 100 - 0.5x^2$

price when $x = 2\sqrt{10} = 100 - 0.5(2\sqrt{10})^2 =$

$= 100 - 0.5(40) = 100 - 20 = \$80$

\$80 will maximize the profit

b) $\bar{C}(x) = \dfrac{C(x)}{x} = 40 + \dfrac{37.5}{x}$

$\bar{C}(2\sqrt{10}) = 40 + \dfrac{37.5}{2\sqrt{10}} = 40 + 5.93 = \$45.93$

VI

$R(x) = 100x - 0.5x^3$

$C(x) = 50x + 37.5$

$P(x) = R(x) - C(x) = 100x - 0.5x^3 - (50x + 37.5)$

$= -0.5x^3 + 50x - 37.5$

$P'(x) = -1.5x^2 + 50$

$-1.5x^2 + 50 = 0$

$-1.5x^2 = -50$

$x^2 = \dfrac{-50}{-1.5} = \dfrac{50}{\frac{3}{2}} = \dfrac{100}{3}$

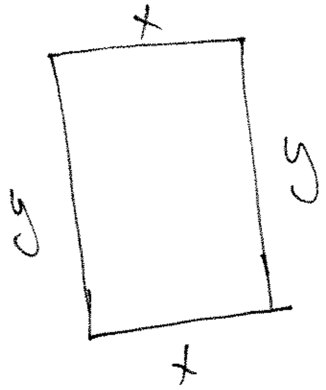
$x = \pm \dfrac{10}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \pm \dfrac{10\sqrt{3}}{3}$

$+ + + + \qquad - - -$

$-\dfrac{10\sqrt{3}}{3} \qquad +\dfrac{10\sqrt{3}}{3}$ max

$p = 100 - 0.5x^2 = 100 - \dfrac{50}{3} = \dfrac{300}{3} - \dfrac{50}{3} = \dfrac{250}{3}$

VII



$2x+2y=200$

Maximize $A=xy$

$2x+2y=200$

$x+y=100$

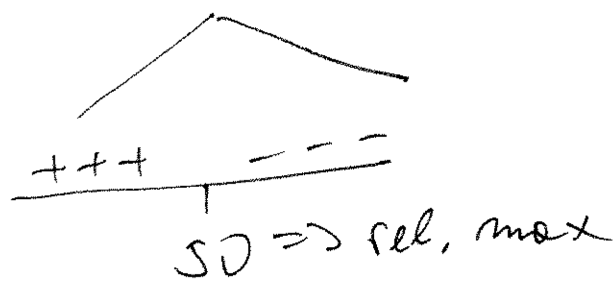
$y=100-x$

$\Rightarrow A=x(100-x)=100x-x^2$

$A'=100-2x$

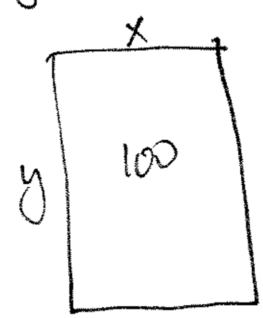
$100-2x=0$

$x=50\,m$



$y=100-x=100-50=50\,m$

VIII



$xy=100$

$P=2x+2y$ -> minimize this

$\frac{xy}{x}=\frac{100}{x} \Rightarrow y=\frac{100}{x}$

-> $x=10$ is rel. min

$y=\frac{100}{x}=\frac{100}{10}=10$

$P=2x+2\frac{100}{x}=2x+\frac{200}{x}=2x+200x^{-1}$

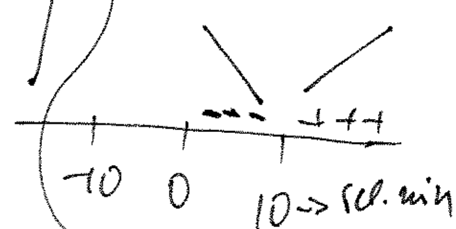
$P'=2-\frac{200}{x^2}$

$x^2\left(2-\frac{200}{x^2}\right)=0\ x^2$ -> $2x^2-200=0$

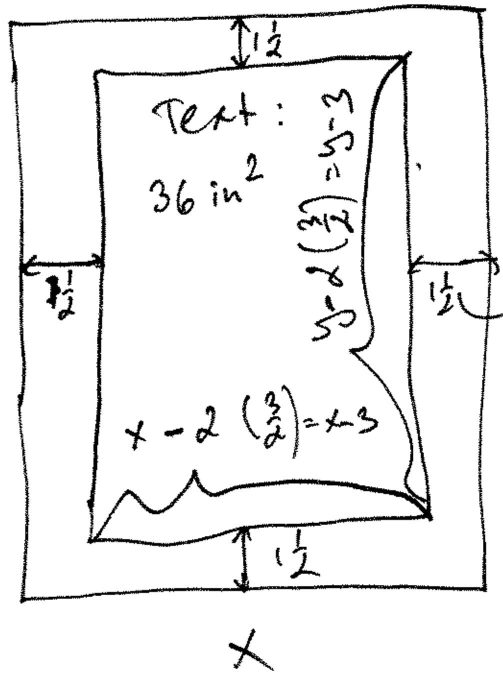
$2x^2=200$

$x^2=100$

$x=\pm 10$



Ex



$1\frac{1}{2} = \frac{3}{2}$ in

Minimize area: $A = xy$

Area of text =

$= (x-3)(y-3) = 36 \text{ in}^2$

$$\Rightarrow \frac{(x-3)(y-3)}{\cancel{x-3}} = \frac{36}{x-3}$$

$$y - 3 = \frac{36}{x-3}$$

$$y = \frac{36}{x-3} + 3$$

$$A = xy = x\left(\frac{36}{x-3} + 3\right) = \frac{36x}{x-3} + 3x$$

$$A' = \frac{(x-3)36 - 36x}{(x-3)^2} + 3$$

$$\frac{36x - 108 - 36x}{(x-3)^2} + 3 = 0$$

$$\cancel{(x-3)^2}\, \frac{-108}{\cancel{(x-3)^2}} = -3(x-3)^2$$

$$-108 = -3(x-3)^2$$

$$(x-3)^2 = \frac{-108}{-3}$$

$$(x-3)^2 = 36$$

$$x - 3 = \pm 6$$

$$x = 9 \qquad x = -3$$

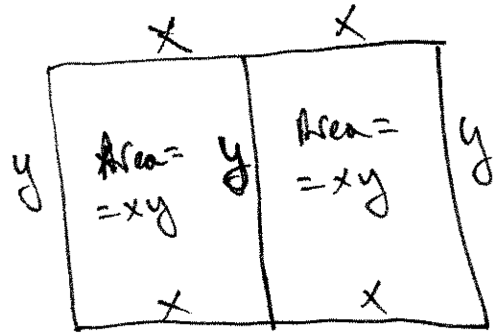
$$y = \frac{36}{x-3} + 3 = \frac{36}{9-3} + 3 = 6 + 3 = 9 \text{ in}$$

--- +++

-3 3 9 → rel min

this makes A' undefined

I



$A = 2xy \rightarrow$ maximize

condition:

$$4x + 3y = 200 \text{ ft}$$

$$\Rightarrow 3y = 200 - 4x$$

$$y = \frac{200 - 4x}{3}$$

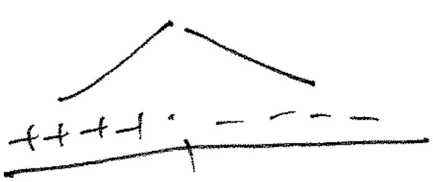
$$A = 2x\,\frac{(200-4x)}{3} = \frac{2}{3}\left(200x - 4x^2\right)$$

$$A' = \frac{2}{3}\left(200 - 8x\right)$$

$$\frac{2}{3}\left(200 - 8x\right) = 0$$

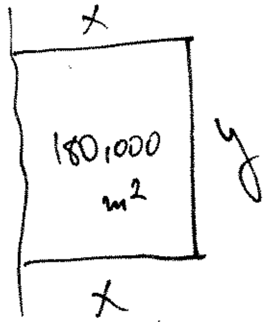
$$200 - 8x = 0$$

$$x = \frac{200}{8} = 25 \text{ ft} \Rightarrow y = \frac{200 - 4x}{3} = \frac{200 - 4(25)}{3} = \frac{100}{3} \text{ ft}$$



$25 \Rightarrow$ a rel. max. (actually it is absolut since A is a downward pointing parabola $\cap$)

XI



Minimise $P = 2x + y$

Condition $xy = 180{,}000$

$\Rightarrow y = \dfrac{180{,}000}{x}$

$$P = 2x + \frac{180{,}000}{x} = 2x + 180{,}000x^{-1}$$

$$P' = 2 - \frac{180{,}000}{x^2}$$

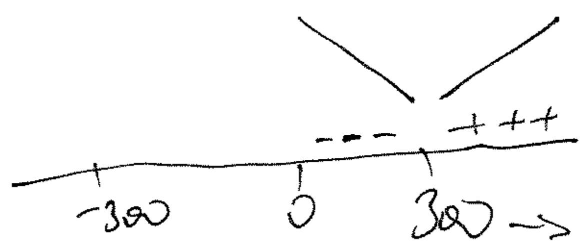
$$x^2\left(2 - \frac{180{,}000}{x^2}\right) = 0 \quad / \times x^2$$

$$2x^2 - 180{,}000 = 0$$

$$2x^2 = 180{,}000$$

$$x^2 = 90{,}000$$

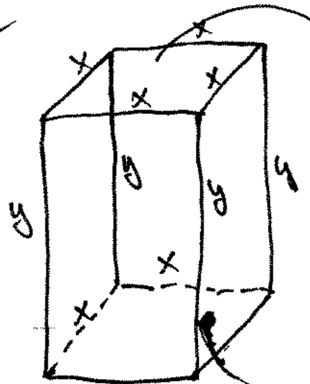
$$x = \pm 300$$



$\rightarrow$ min.

$$y = \frac{180{,}000}{x} = \frac{180{,}000}{300} = 600\text{ m}$$

XII



Maximize $V = x^2 y$

Condition:

$2x^2 + 4xy = 150 \text{ in}^2$

$$\frac{4xy}{4x} = \frac{150 - 2x^2}{4x}$$

$$y = \frac{150 - 2x^2}{4x}$$

$$V = x^{\cancel{2}} \frac{150 - 2x^2}{4\cancel{x}} = \frac{1}{4}\left(150x - 2x^3\right)$$

$$V' = \frac{1}{4}\left(150 - 6x^2\right)$$

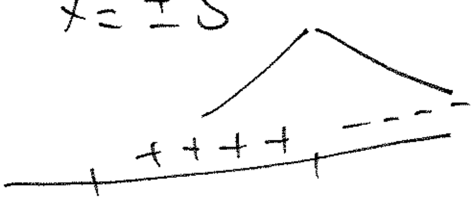
$$\frac{1}{4}\left(150 - 6x^2\right) = 0$$

$$150 - 6x^2 = 0$$

$$x^2 = \frac{150}{6}$$

$$x^2 = 25$$

$$x = \pm 5$$



$$y = \frac{150 - 2x^2}{4x} = \frac{150 - 2(5)^2}{4(5)} = \frac{150 - 50}{20} = 5 \text{ in}$$

$\overline{\text{XIV}}$ Sketch the graph;

1) $f(x) = \dfrac{x^2}{(x+1)^2} = \dfrac{x^2}{x^2+2x+1}$

Domain $x \neq -1$

Range : $y \geq 0$
because of the square powers

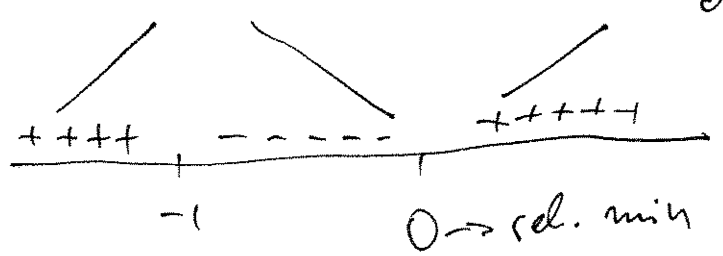
HA : $y = 1$

V.A. $x = -1$

$$f'(x) = \frac{(x+1)^2 2x - x^2 2(x+1)}{(x+1)^4} = \frac{\cancel{(x+1)}(\cancel{2x^2}+2x-\cancel{2x^2})}{(x+1)^{\cancel{4}\,3}}$$

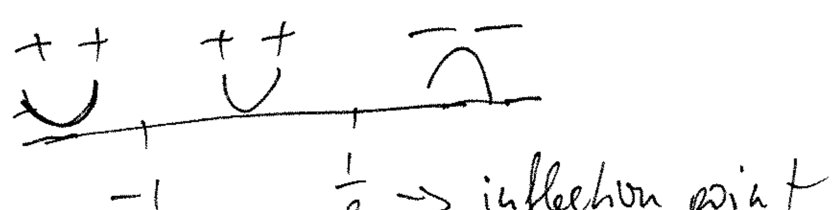
$$= \frac{2x}{(x+1)^3}$$

→ critical values : $x=0$ ($x=-1$ is an asymptote)
but you still test around it



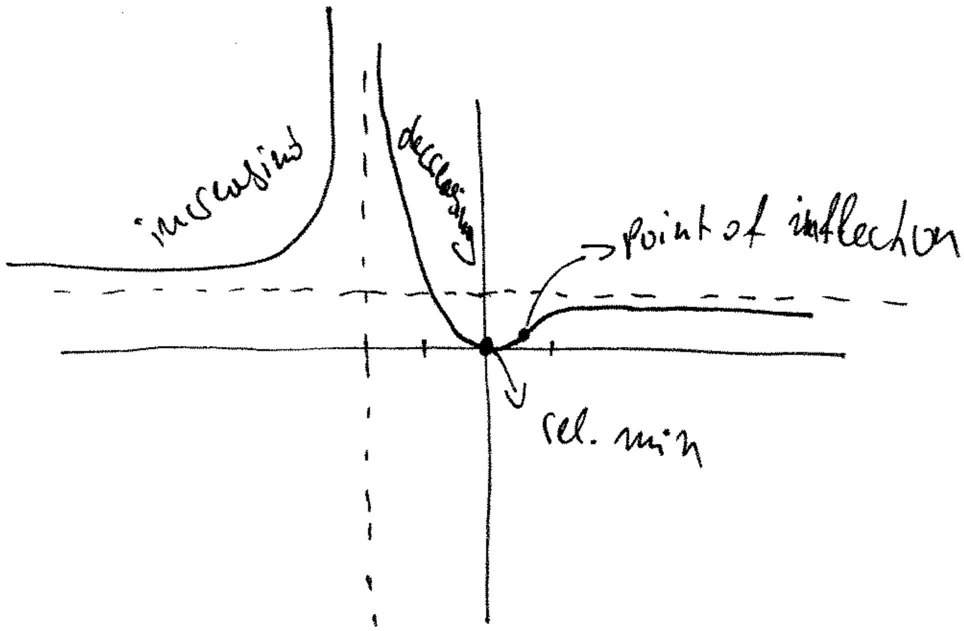
$$f'' = \frac{(x+1)^3 2 - 2x\,3(x+1)^2}{(x+1)^6} = \frac{\cancel{(x+1)}^2(2x+2-6x)}{(x+1)^{\cancel{6}}} =$$

$$= \frac{2-4x}{(x+1)^4}$$



$f(0) = 0$

$f(\frac{1}{2}) = \dfrac{\frac{1}{4}}{\frac{9}{4}} = \dfrac{1}{9}$



2) $f(x)= \dfrac{2x-1}{x+2}$ Domain : $x \neq -2$

H.A. : $y=2$

V.A. : $x=-2$

$$f'(x)= \frac{(x+2)(2)-(2x-1)(1)}{(x+2)^2} = \frac{2x+4-2x+1}{(x+2)^2} = \frac{5}{(x+2)^2} = 5(x+2)^{-2}$$

$+++$ | $++++$

-2

no rel. max/min

$$f''(x) = \frac{-10}{(x+2)^3}$$

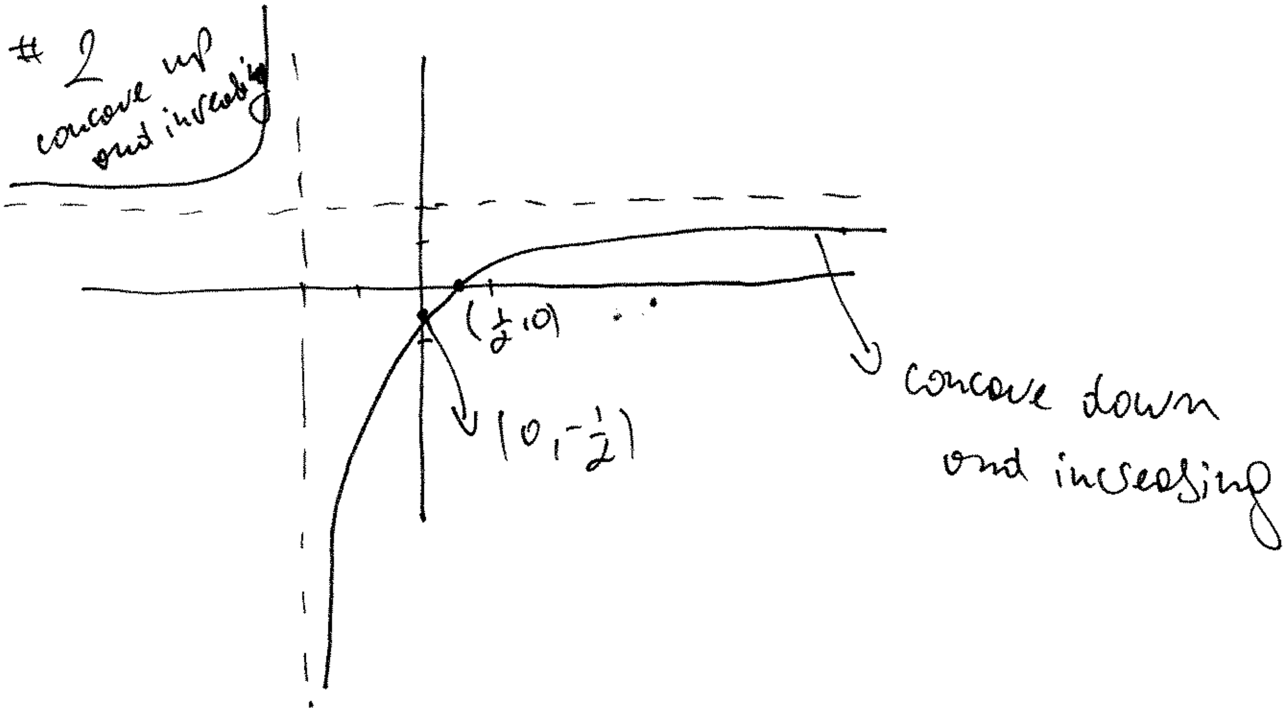
$+\ +$ $\cup$ | $-\ -$ $\cap$

-2

~~x~~ intercept : $2x-1=0 \Rightarrow x=\frac{1}{2}$

y intercept $f(0)=-\frac{1}{2}$

2



3 $f(x) = \dfrac{x^2+3}{1-x}$ $\Rightarrow$ Domain $x \neq 1$

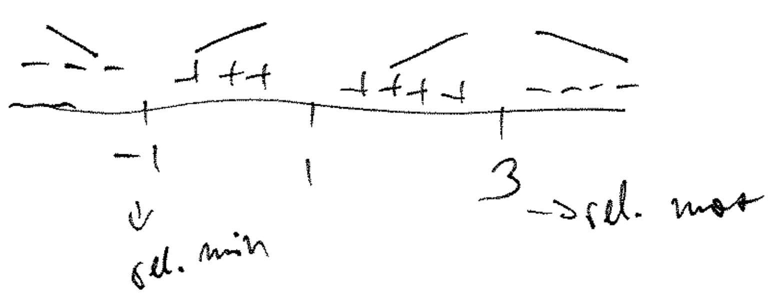
H.A: none

V.A. $x = 1$

$$f'(x) = \frac{(1-x)2x - (x^2+3)(-1)}{(1-x)^2} = \frac{2x - 2x^2 + x^2 + 3}{(1-x)^2}$$

$$= \frac{-x^2+2x+3}{(1-x)^2} = \frac{-(x^2-2x-3)}{(1-x)^2} = \frac{-(x-3)(x+1)}{(1-x)^2}$$

$\Rightarrow$ critical values $x=-1$ $x=3$; $x=1$ is asymptote

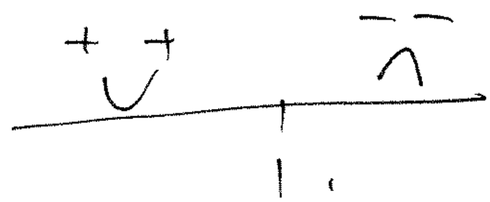


$$f''(x) = \frac{(1-x)^2(-2x+2) - (\cancel{-x}^2+2x+3)\,2(1-x)(-1)}{(1-x)^4}$$

$$= \frac{\cancel{(1-x)}\left((1-x)(-2x+2) + \left(-2x^2+4x+6\right)\right)}{(1-x)^{\cancel{4}\,3}}$$

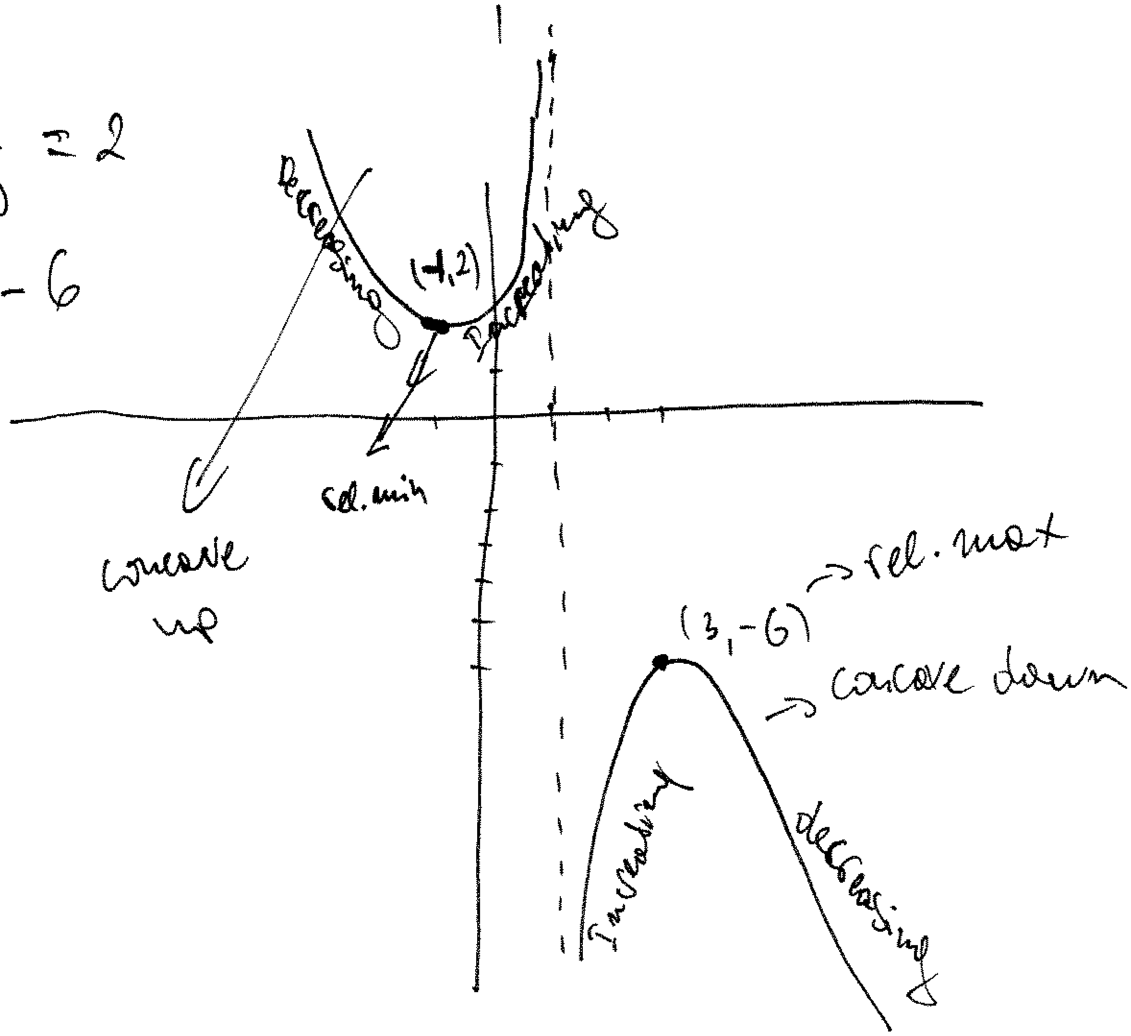
$$= \frac{-\cancel{2x}+2+\cancel{2x^2}-2x-\cancel{2x^2}+\cancel{4x}+6}{(1-x)^3}$$

$$= \frac{8}{(1-x)^3}$$



$$f(-1) = \frac{1+3}{1-(-1)} = 2$$

$$f(3) = \frac{12}{-2} = -6$$



#4 $y = \frac{x^2+1}{x^2-2}$ — Domain: $x^2-2 \neq 0$

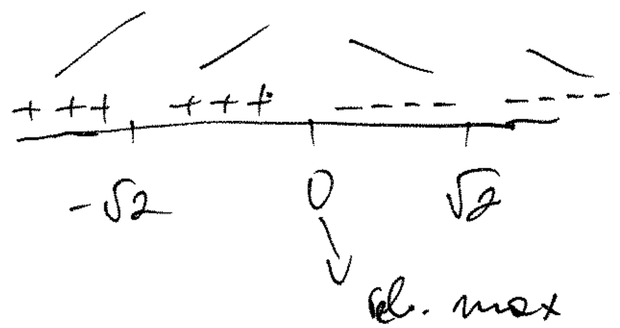
$x \neq \pm\sqrt{2}$

H.A.: $y = 1$

V.A.: $x^2-2=0$ $\quad x^2=2 \Rightarrow x = \pm\sqrt{2}$

$$y' = \frac{(x^2-2)(2x) - (x^2+1)2x}{(x^2-2)^2} = \frac{\cancel{2x^3}-4x-\cancel{2x^3}-2x}{(x^2-2)^2}$$

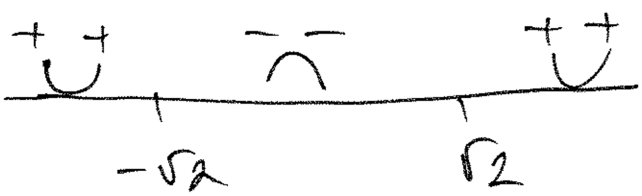
$$= \frac{-6x}{(x^2-2)^2}$$



$$y'' = \frac{(x^2-2)^2(-6) - (-6x)2(x^2-2)2x}{(x^2-2)^4} =$$

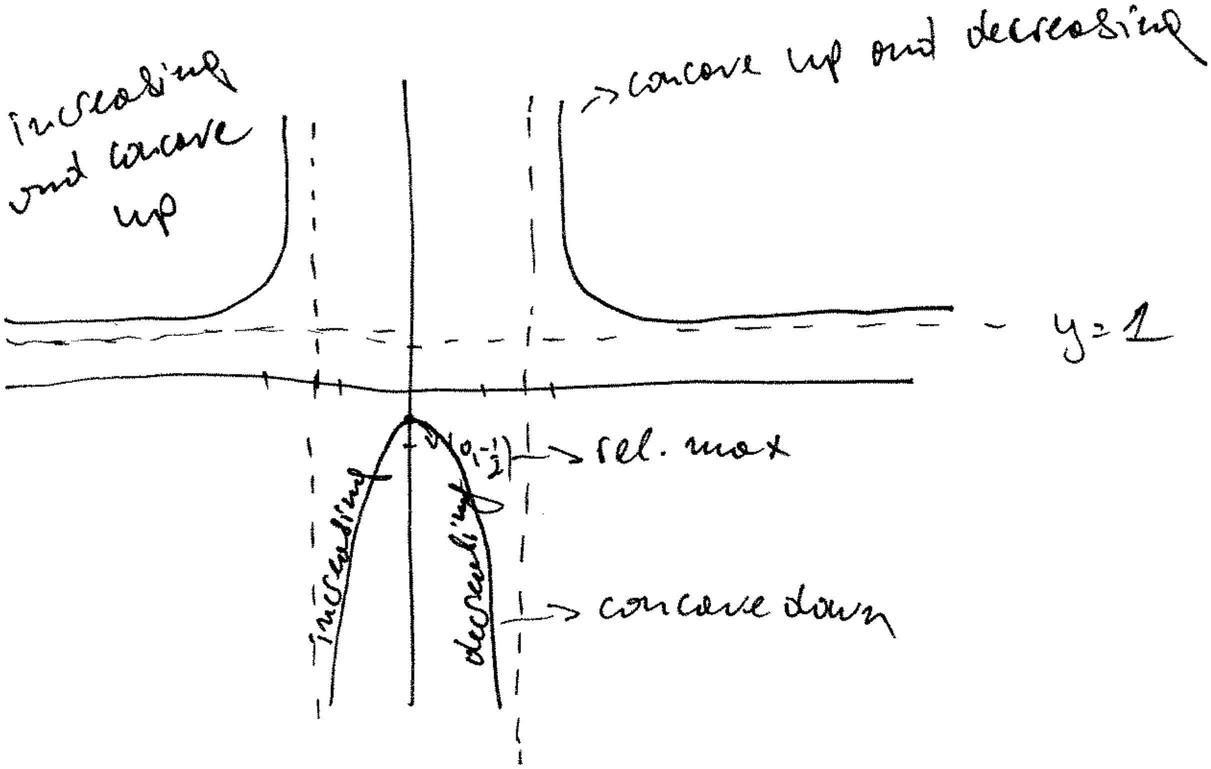
$$= \frac{\cancel{(x^2-2)}(-6x^2+12+24x^2)}{(x^2-2)^{\cancel{4}\,3}} = \frac{18x^2+12}{(x^2-2)^3}$$

$18x^2+12=0 \Rightarrow x^2 = -\frac{12}{18} \Rightarrow$ no real solutions



$f(0) = -\frac{1}{2}$

#4



increasing and concave up
concave up and decreasing
y = 1
(0, -1/2)
rel. max
increasing
decreasing
concave down

XV

1) $y' = 2x \ln(2x+3) + \frac{2x^2}{2x+3}$

2) $y' = \frac{x \cdot \frac{1}{x} - (1+\ln x)(1)}{x^2} = \frac{1-1-\ln x}{x^2} = -\frac{\ln x}{x^2}$

3) $y = \ln\left(\frac{t^2+3}{\sqrt{1-t}}\right) = \ln(t^2+3) - \ln\sqrt{1-t} =$

$= \ln(t^2+3) - \frac{1}{2}\ln(1-t)$

$y' = \frac{2t}{t^2+3} - \frac{1}{2}\frac{1}{1-t}(-1) = \frac{2t}{t^2+3} + \frac{1}{2(1-t)}$

4) $y = 3\ln x + \frac{1}{2}\ln(x+1)$

$y' = \frac{3}{x} + \frac{1}{2(x+1)}$

5) $y = \frac{1}{4}\ln\left(\frac{3x+2}{x^2-5}\right) =$

$= \frac{1}{4}\left(\ln(3x+2) - \ln(x^2-5)\right) = \frac{1}{4}\ln(3x+2) - \frac{1}{4}\ln(x^2-5)$

$y' = \frac{3}{4(3x+2)} - \frac{2x}{4(x^2-5)} = \frac{3}{4(3x+2)} - \frac{x}{2(x^2-5)}$

6) $y = 3\ln t + \ln(t^2-1)$

$y' = \frac{3}{t} + \frac{2t}{t^2-1}$

XV 7) $y' = 4e^x + 4xe^x$

8) $y' = \dfrac{(1+e^{2x})(1) - x(2e^{2x})}{(1+e^{2x})^2} = \dfrac{1+e^{2x} - 2xe^{2x}}{(1+e^{2x})^2}$

9) $y = e^{\sqrt{x^2-9}} = e^{(x^2-9)^{1/2}}$

$y' = e^{(x^2-9)^{1/2}}\left(\frac{1}{\cancel{2}}(x^2-9)^{-1/2}(\cancel{2}x)\right) = \dfrac{x}{\sqrt{x^2-9}}\, e^{\sqrt{x^2-9}}$

10) $y' = 2e^{2x}$ (since e^3 is a constant)

11) $y = 2e^{-2x} + \frac{1}{2}e^{2x}$

$y' = -4e^{-2x} + \frac{1}{\cancel{2}}e^{2x}\cancel{2} = \dfrac{-4}{e^{2x}} + e^{2x}$

12) $y' = -2e^{-x^3}(-3x^2) = 6x^2e^{-x^3}$

XVI a) $x - \dfrac{dy}{dx} = 6 \Rightarrow \dfrac{dy}{dx} = x - 6$

b) $4x^2\dfrac{dy}{dx} + 8xy + \dfrac{3}{y^2}\dfrac{dy}{dx} = 0$

$\left(4x^2 + \frac{3}{y^2}\right)\dfrac{dy}{dx} = -8xy$

$\dfrac{dy}{dx} = \dfrac{-8xy}{4x^2 + \frac{3}{y^2}}$

c) $y^2 + 2xy\frac{dy}{dx} + 4x\frac{dy}{dx} + 4y = 0$

$$(2xy + 4x)\frac{dy}{dx} = -4y - y^2$$

$$\frac{dy}{dx} = \frac{-4y - y^2}{2xy + 4x}$$

d) $\frac{xy - y^2}{y - x} = 1$ multiply both sides by $(y-x)$ or use quotient rule

If you use quotient rule

$u = xy - y^2$ $v = y - x$

$u' = y + x\frac{dy}{dx} - 2y\frac{dy}{dx}$ $v' = \frac{dy}{dx} - 1$

$$\frac{vu' - uv'}{v^2} = \frac{(y-x)\left(y + x\frac{dy}{dx} - 2y\frac{dy}{dx}\right) - (xy - y^2)\left(\frac{dy}{dx} - 1\right)}{(y-x)^2}$$

$$= 0$$

$$\Rightarrow (y-x)\left(y + x\frac{dy}{dx} - 2y\frac{dy}{dx}\right) - (xy - y^2)\left(\frac{dy}{dx} - 1\right) = 0$$

$$y^2 + \cancel{xy\frac{dy}{dx}} - 2y^2\frac{dy}{dx} - \cancel{xy} - x^2\frac{dy}{dx} + 2xy\frac{dy}{dx}$$

$$- \cancel{xy\frac{dy}{dx}} + y^2\frac{dy}{dx} + \cancel{xy} - \cancel{y^2} = 0$$

$$\cancel{xy} \quad -y^2\frac{dy}{dx} - x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = 0$$

$$\Rightarrow (-y^2 - x^2 + 2xy)\frac{dy}{dx} = 0$$

$$\text{or } \frac{dy}{dx} = 0$$

You can see this also by

$$\frac{xy - y^2}{y-x} = \frac{-y(-x+y)}{y-x} = \frac{-y(\cancel{y-x})}{\cancel{y-x}} = 1$$

$$\Rightarrow -y = 1$$

$$y = -1$$

$$\text{or } \frac{dy}{dx} = 0$$

e) $\dfrac{2x+y}{x-5y} = 1$ Multiply both sides by $(x-5y)$

$2x+y = x-5y$

$2+\dfrac{dy}{dx} = 1-5\dfrac{dy}{dx}$

$\dfrac{6dy}{dx} = -1$

$\dfrac{dy}{dx} = -\dfrac{1}{6}$

f) $2xy + x^2\dfrac{dy}{dx} = e^{x+y}\left(1+\dfrac{dy}{dx}\right)$

$x^2\dfrac{dy}{dx} = -2xy + e^{x+y} + e^{x+y}\dfrac{dy}{dx}$

$x^2\dfrac{dy}{dx} - e^{x+y}\dfrac{dy}{dx} = -2xy + e^{x+y}$

$\dfrac{(x^2-e^{x+y})}{(x^2-e^{x+y})}\dfrac{dy}{dx} = \dfrac{-2xy+e^{x+y}}{x^2-e^{x+y}}$

$\dfrac{dy}{dx} = \dfrac{-2xy+e^{x+y}}{x^2-e^{x+y}}$

g) $1 - e^{y} - xe^{y}\dfrac{dy}{dx} = 0$

$-xe^{y}\dfrac{dy}{dx} = -1 + e^{y}$

$\dfrac{dy}{dx} = \dfrac{-1+e^{y}}{-xe^{y}}$

h) $\ln(xy) = 2 \Rightarrow \ln x + \ln y = 2$

$\dfrac{1}{x} + \dfrac{1}{y}\dfrac{dy}{dx} = 0$

$\dfrac{1}{y}\dfrac{dy}{dx} = -\dfrac{1}{x}$

$\dfrac{dy}{dx} = -\dfrac{y}{x}$

i) $\ln(2x+3y) = 7$

$\dfrac{1}{2x+3y}\left(2 + 3\dfrac{dy}{dx}\right) = 0$

$2 + 3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = -\dfrac{2}{3}$

XVIII

a) $x^2+y^2=9$

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x}{y}\bigg|_{(0,3)}=-\frac{0}{3}=0$$

$$y=mx+b=0x+b$$

$$3=(0)(0)+b$$

$$b=3 \Rightarrow y=3$$

b) $4x\frac{dy}{dx}+4y+2x=0$

$$4x\frac{dy}{dx}=-4y-2x$$

$$\frac{dy}{dx}=\frac{-4y-2x}{4x}\bigg|_{(1,1)}=\frac{-6}{4}=-\frac{3}{2}$$

$$y=mx+b \qquad y=-\frac{3}{2}x+b$$

$$1=-\frac{3}{2}(1)+b$$

$$1=-\frac{3}{2}+b$$

$$\frac{2}{2}+\frac{3}{2}=b$$

$$b=\frac{5}{2}$$

$$y=-\frac{3}{2}x+\frac{5}{2}$$