Review Questions Solutions

I

a) $\lim\limits_{x\to -3} \dfrac{2x^2+5x-3}{x^2-9} = \lim\limits_{x\to -3} \dfrac{(2x-1)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}}$

$= \lim\limits_{x\to -3} \dfrac{2x-1}{x-3} = \dfrac{-7}{-6} = \dfrac{7}{6}$

b) $\lim\limits_{x\to 5} \dfrac{x^2-3x-3}{x^2-8x+1} = \dfrac{25-15-3}{25-40+1} = \dfrac{7}{-14} = \dfrac{-1}{2}$

c) $\lim\limits_{x\to -\frac{3}{4}} \dfrac{4x}{4x+3} = \dfrac{-3}{-3+3} = \dfrac{-3}{0} \Rightarrow$ DNE

d) In class

e) $\lim\limits_{x\to -1} \dfrac{2x}{x+1} = \dfrac{-2}{-1+1} = \dfrac{-2}{0} \Rightarrow$ DNE

f) $\lim\limits_{x\to -5} \dfrac{x^2+8x+15}{x^2+5x} = \lim\limits_{x\to -5} \dfrac{\cancel{(x+5)}(x+3)}{x\cancel{(x+5)}} = \dfrac{-2}{-5} = \dfrac{2}{5}$

g) $\lim\limits_{x\to 5^-} f(x) = 7(5)-10 = 25$

$\lim\limits_{x\to 5^+} f(x) = 25$

$\Big\}\ \lim\limits_{x\to 5} f(x) = 25$

h) $\lim\limits_{x \to 2^-} f(x) = \dfrac{2^3-4}{2-3} = \dfrac{4}{-1} = -4$

$\lim\limits_{x \to 2^+} f(x) = \dfrac{3-2^2}{2} = -\dfrac{1}{2}$

$\Rightarrow$ DNE

i) $\lim\limits_{x \to -\infty} \dfrac{4}{x^2-2x} = \lim\limits_{x \to -\infty} \dfrac{\frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2}} = \lim\limits_{x \to -\infty} \dfrac{\frac{4}{x^2}}{1-\frac{2}{x}} = \dfrac{0}{1-0} = 0$

j) $\lim\limits_{x \to -\infty} \dfrac{3x^2+2}{x^2-4} = \lim\limits_{x \to -\infty} \dfrac{\frac{3x^2}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \lim\limits_{x \to -\infty} \dfrac{3+\frac{2}{x^2}}{1-\frac{4}{x^2}} = 3$

k) $\lim\limits_{x \to \infty} \dfrac{4x^2+5x}{x^2-4x+1} = \lim\limits_{x \to \infty} \dfrac{\frac{4x^2}{x^2} + \frac{5x}{x^2}}{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{1}{x^2}} = \lim\limits_{x \to \infty} \dfrac{4+\frac{5}{x}}{1-\frac{4}{x}+\frac{1}{x^2}} = \dfrac{4+0}{1-0+0}$

$= 4$

l) $\lim\limits_{x \to -\infty} \dfrac{5x^3-8}{4x^2+5x} = \lim\limits_{x \to -\infty} \dfrac{\frac{5x^3}{x^3} - \frac{8}{x^3}}{\frac{4x^2}{x^3} + \frac{5x}{x^3}} = \lim\limits_{x \to -\infty} \dfrac{5-\frac{8}{x^3}}{\frac{4}{x} - \frac{5}{x^2}}$

$= \dfrac{5-0}{0-0} = \dfrac{5}{0} \Rightarrow$ DNE

II a), b) in class

c) $x^2-4x+4=0$

$(x-2)^2=0 \Rightarrow x=2 \Rightarrow$ Discontinuity @ $x=2$

d) $x^2+5x=0$

$x(x+5)=0 \Rightarrow$ Discontinuities @

$x=0 \quad x=-5$

e) $x^2-6x+5=(x-1)(x-5)=0 \Rightarrow$

Discontinuities @ $x=1$ $x=5$

f) $\lim_{x\to 1^-} g(x)=2 \qquad \lim_{x\to 1^+} g(x)=1$

$\lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x) \Rightarrow$ ~~DNE~~

$\Rightarrow \lim_{x\to 1} g(x)$ DNE $\Rightarrow$ Discontinuity @ $x=1$

g) $\lim_{x\to 1^-} f(x) = 1^3+1=2 \qquad \lim_{x\to 1^+} f(x)=2$

$\left.\begin{array}{l} \Rightarrow \lim_{x\to 1} f(x)=2 \\ f(1)=1^3+1=2 \end{array}\right\} \Rightarrow \lim_{x\to 1} f(x) = f(1) \Rightarrow$

$\Rightarrow$ Continuous everywhere

hw

III a) $f'(x) = \lim_{h \to 0} \dfrac{(x+h)^2 + (x+h) + 3 - (x^2+x+3)}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{x^2} + 2xh + h^2 \cancel{+x} + h \cancel{+3} \cancel{-x^2} \cancel{-x} \cancel{-3}}{h} =$

$= \lim_{h \to 0} \dfrac{\cancel{h}(2x+h+1)}{\cancel{h}} = \lim_{h \to 0} 2x+h+1 = 2x+1$

b) In class

c) $\lim_{h \to 0} \dfrac{(x+h)^3 + 3 - (x^3+3)}{h} =$

$= \lim_{h \to 0} \dfrac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{+3} \cancel{-x^3} \cancel{-3}}{h} =$

$= \lim_{h \to 0} \dfrac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim_{h \to 0} 3x^2 + 3xh + h^2 = 3x^2$

d) $\lim_{h \to 0} \dfrac{(x+h)^3 + 2(x+h) + 1 - (x^3+2x+1)}{h} =$

$= \lim_{h \to 0} \dfrac{\cancel{x^3} + 3xh^2 + 3x^2h + h^3 \cancel{+2x} + 2 \cdot h \cancel{+1} \cancel{-x^3} \cancel{-2x} \cancel{-1}}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{h}(3xh + 3x^2 + h^2 + 2)}{\cancel{h}} = \lim_{h \to 0} 3xh + 3x^2 + h^2 + 2$

$= 3x^2 + 2$

IV a) $y = \frac{1}{3}x^3 - 3x^{-3}$

$y' = x^2 + 9x^{-4}$

b) $y = x^{1/2} + x^{2/3} - x^{-1/2} + x^{-2/3}$

$y' = \frac{1}{2}x^{-1/2} + \frac{2}{3}x^{-1/3} + \frac{1}{2}x^{-3/2} - \frac{2}{3}x^{-5/3}$

c) $u = 3x - 1$ $\quad v = x^2 - 4x$

$u' = 3$ $\quad v' = 2x - 4$

$f'(x) = uv' + vu' = (3x-1)(2x-4) + 3(x^2 - 4x)$

d) $u = x^2 + 1$ $\quad v = 3x^3 + 1$

$u' = 2x$ $\quad v' = 9x^2$

$y' = (x^2+1)\,9x^2 + (3x^3+1)\,2x$

e) $p = \frac{\overbrace{2q-1}^{u}}{\underbrace{q^2}_{v}}$ $\quad u' = 2$, $v' = 2q$

$$\frac{dp}{dq} = \frac{vu' - uv'}{v^2} = \frac{q^2(2) - (2q-1)2q}{q^4} = \frac{2q^2 - 4q^2 + 2q}{q^4}$$

$$= \frac{-2q^2 + 2q}{q^4} = \frac{-2(q-1)}{q^3}$$

f) $h(x) = \dfrac{x^{1/2}}{3x+1}$

$u = x^{1/2}$ $\quad v = 3x+1$

$u' = \frac{1}{2}x^{-1/2}$ $\quad v' = 3$

$$h'(x) = \frac{vu' - uv'}{v^2} = \frac{(3x+1)\frac{1}{2}x^{-1/2} - 3x^{1/2}}{(3x+1)^2} =$$

g) $u = 5x^4 - 2x^2 + 1$ $\quad v = x^3 + 1$

$u' = 20x^3 - 4x$ $\quad v' = 3x^2$

$$c'(x) = \frac{vu' - uv'}{v^2} = \frac{(x^3+1)(20x^3 - 4x) - (5x^4 - 2x^2 + 1)3x^2}{(x^3+1)^2}$$

h) $y' = 3(x^3 - 4x^2)^2(3x^2 - 8x)$

i) $y' = 6(5x^6 + 6x^4 + 5)^5(30x^5 + 24x^3)$

j) $g(x) = (x^3 - 4x)^{-1/2}$

$g'(x) = -\frac{1}{2}(x^3 - 4x)^{-3/2}(3x^2 - 4)$

k) $f(x) = \underbrace{x^2}_{u}\underbrace{(2x^4+5)^8}_{v}$

$u = x^2$ $\quad v = (2x^4+5)^8$

$u' = 2x$

$v' = 8(2x^4+5)^7 8x^3$

$= 64x^3(2x^4+5)^7$

$$f'(x) = uv' + vu' = x^2 64x^3(2x^4+5)^7 + 2x(2x^4+5)^8$$

$$= 64x^5(2x^4+5)^7 + 2x(2x^4+5)^8$$

l) $S(x) = \dfrac{\overbrace{(3x+1)^2}^{u}}{\underbrace{x^2-4}_{v}}$ $\qquad u' = 2(3x+1)\,3 = 6(3x+1)$

$\qquad\qquad\qquad\qquad v' = 2x$

$$S'(x) = \frac{vu' - uv'}{v^2} = \frac{(x^2-4)\,6(3x+1) - (3x+1)^2\,2x}{(x^2-4)^2}$$

$$= \frac{6(3x+1)(x^2-4) - 2x(3x+1)^2}{(x^2-4)^2}$$

m), n) in class

o) $u = x \qquad v = \sqrt{x^2-4} = (x^2-4)^{1/2}$

$\quad u' = 1 \qquad v' = \frac{1}{\cancel{2}}(x^2-4)^{-1/2}(\cancel{2}x)$

$$y' = uv' + vu' = x\,x(x^2-4)^{-1/2} + (x^2-4)^{1/2}$$

$$= x^2(x^2-4)^{-1/2} + (x^2-4)^{1/2}$$

p) in class

V) a) in class

b) $y' = \frac{1}{4}(1-x)^5(-1) = -\frac{1}{4}(1-x)^5$

$y'' = -\frac{5}{4}(1-x)^4(-1) = \frac{5}{4}(1-x)^4$

$y''' = 5(1-x)^3(-1) = -5(1-x)^3$

b) $y^{IV} = -15(1-x)^2(-1) = 15(1-x)^2$

$y^{V} = 30(1-x)(-1) = -30 + 30x$

c) $y = (x^2-4)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(x^2-4)^{-1/2}(2x) = \underbrace{x}_{u}\,\underbrace{(x^2-4)^{-1/2}}_{v}$$

$$u' = 1 \qquad v' = -\frac{1}{2}(x^2-4)^{-3/2}(2x)$$

$$= -x(x^2-4)^{-3/2}$$

$$\frac{d^2y}{dx^2} = uv' + vu' = -x^2(x^2-4)^{-3/2} + (x^2-4)^{-1/2}$$

d) $$\frac{d^2y}{dx^2} = \frac{\overbrace{x}^{u}}{\underbrace{x^2+1}_{v}} \qquad u' = 1 \qquad v' = 2x$$

$$\frac{d^3y}{dx^3} = \frac{vu' - uv'}{v^2} = \frac{x^2+1-2x^2}{x^2+1} = \frac{\overbrace{1-x^2}^{u}}{\underbrace{x^2+1}_{v}}$$

$$\frac{d^4y}{dx^4} = \frac{(x^2+1)(-2x) - (1-x^2)2x}{(x^2+1)^2} \qquad u' = -2x$$

$$v' = 2x$$

$$= \frac{\cancel{-2x^3} - 2x - 2x + \cancel{2x^3}}{(x^2+1)^2} = \frac{-4x}{(x^2+1)^2}$$

$\boxed{105}$

$u = 60x^2 \qquad v = 2x+1$

$u' = 120x \qquad v' = 2$

$$\overline{MR} = R'(x) = \frac{vu' - uv'}{v^2} = \frac{120x(2x+1) - 120x^2}{(2x+1)^2}$$

$$= \frac{120x^2 + 120x}{(2x+1)^2}$$

$\boxed{98}$

$$v(t) = \frac{ds}{dt} = 140 + 4t^{-1/2}$$

$$\text{acceleration} = \frac{d^2s}{dt^2} = -2t^{-3/2} = -\frac{2}{\sqrt{t^3}}$$

$$= -\frac{2}{\sqrt{64}} = -\frac{2}{8} = -\frac{1}{4} \text{ ft/sec}^2$$